Value at Risk: An analysis for the European Stock Exchanges

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I hereby declare that the work submitted is mine and that where I have made use of another’s work, I have attributed the source(s) according to the Regulations set in the Student’s Handbook.

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Abstract

This dissertation focuses on the estimation of Value at Risk in six European Stock Exchanges from the beginning of the millennium. It presents the theoretical framework regarding the VaR techniques as well as the ARCH models which are commonly used in the estimation of market risk. On the empirical part, the dissertation provides an insight into parametric models like Risk Metrics and non parametric like Historical Simulation and in order to evaluate their predictive ability during the recent global financial crisis they are backtested. In addition, models of the ARCH family are being presented extensively since they are commonly used in the VaR forecasting procedure. The Akaike’s Information as well as the Schwarz’s Bayesian Information Criterion are examined so as to be concluded if the aforementioned models are trustworthy and could predict VaR accurately.

Keywords: Value at Risk, Backtesting, ARCH, AIC, SBIC

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Introduction

Risk, is the volatility of unexpected outcomes, which can represent the value of assets, equity or earnings (Jorion, 2007). The latest global financial crisis in 2008, is just another example of how unstable and liquid the conditions on both domestic and international level are, and pointed out the vulnerability of the global Stock Exchanges to market risk. According to the European Banking Authority “market risk can be defined as the risk of losses in on and off-balance sheet positions arising from adverse movements in market prices or market rates”

The burst of the global financial crisis back in 2008 revealed that both the corporations as well as the nations themselves were unprepared to deal with the new conditions that appeared. Many economies around the world fell in recession, with some European countries to face the most severe consequences. Countries like Ireland, Iceland and Portugal faced a dramatic shrinkage of their economy and a rescue package from EU, IMF and ECB was necessary in order to avoid the default. On the other hand there were countries within EU that didn’t undergo those vast difficulties, like UK, France and Germany. These economies remain stable regardless of the condition in the rest European states. The sui generis phenomenon that took place in Europe with the implementation of the Common Market increases the interrelationship among European economies but, as the crisis pointed out, not all countries have suffered to the same extent. This dissertation aims to investigate if the condition of the country's economy has played a major role in the minimization of the crisis’ consequences. That is why the three richest and three of the countries that participated in a rescue program are selected for the research. In addition it will be examined if the commonly used VaR models were accurate in forecasting the market risk of the European Stock Exchanges during the crisis. In order to do so, parametric models like Risk Metrics and non parametric like Historical Simulation were estimated and backtested. Apart from these, many models from the ARCH family were estimated so as to decide which model could have forecasted better VaR in the years followed 2008. Finding an accurate model could prove to be very beneficial in the future, since the countries could use the models as one more tool towards predicting market risk and eliminate the unpleasant consequences of a crisis.
1. Value at Risk

The concept of VaR models was developed when it was apparent that mismanagement and misinterpretations of financial risk could lead to financial disasters. Financial Risk management refers to the design and implementation of procedures for identifying, measuring and managing financial risk (Jorion 2007) and VaR models can be a useful tool.

VaR is a statistic of the dispersion of a distribution and refers to a portfolio’s worst outcome likely to occur over a predetermined period and a given confidence level (Angelidis, Benos and Degiannakis 2006). It is a single number, expressed on a currency unit that reveals an institution’s exposure to market risk, where market risk is the risk that arises due to changes in market prices. This conceptual simplicity is the reason why VaR has gained popularity and has been imposed by the Basel Committee on Banking Supervision (1996) as the tool in order to calculate market risk.

VaR is expressed as a $100\alpha\%$ quantile of the density of returns and can be interpreted as follows: At time $t$, the probability that the return of a portfolio will be lower than the VaR equals to $100\alpha\%$ and the mathematical expression of the above sentence is given by the equation (1):

$$
\text{VaR}_t^{\alpha} = \sup \{ r \mid P[R_t \leq r] \leq \alpha \}
$$

On the years that followed, the VaR methodology was evolved in order to measure other types of risks apart from that of market like liquidity risk (L-Var). During the financial crisis, it was apparent that understanding and measuring the liquidity risk was vital not only in order to forecast but also to impugn the crisis. Liquidity risk takes the form of either market liquidity risk which arises from the assets illiquidity or the funding cash flow liquidity risk that arises when the liabilities can be paid fully on in time. But although VaR could also be a useful tool towards dealing with financial crisis, it doesn’t pay attention to the results when a position is liquidated. Therefore L VaR was introduced as a tool in order to account liquidity risk which is inherent to all markets.

VaR analysis should be implemented not only to financial institutions but also to countries as a whole since, due to the globalization that exists today, the results of one
country could provide useful information for others. A good example could be Europe; the European economies are closely connected and integrated due to the Common Market and the single currency. As a result, a VaR analysis for a European country could be proved beneficial for another EU member.

1.1 VaR Methodologies

There are many methodologies and models in order to calculate VaR that has been under scrutiny by many researchers.

The models that are used in order to calculate VaR are classified into three categories.

- The parametric models such as the Risk Metrics and GARCH
- The non parametric models such as the Historical Simulation
- The semi parametric models such as Filtered Historical Simulation and Extreme Value Theory

2. Parametric models

The first category, the parametric models, proposes a specific parameterization for the behavior of prices. Risk Metrics make the assumption that the sample under scrutiny follows the normal distribution and that tomorrow’s volatility is affected by the weighted average of today’s volatility plus today’s squared return. The variance, that is the volatility, is calculated with the use of the Exponentially Weighted Moving Average (EWMA) according to the following equation (2):

$$\sigma^2_{t+1} = \lambda \cdot \sigma^2_t + (1-\lambda) \cdot R^2_t$$  \hspace{1cm} (2)

where:

- $\sigma^2_{t+1}$: is the conditional covariance of the day t+1
- $\sigma^2_t$: is the conditional covariance of the day t (the previous day)
- $\lambda$: is a parameter set to be equal to 0,94 for large data
- $R^2_t$: is the squared return of the Index on day t
The unknown parameter $\lambda$ is set to be 0.94 since as the sample increases in size the estimates are resembled to each other. The Risk Metric model was introduced by J.P Morgan (1996) and it has its roots on the Integrated GARCH (IGARCH) model according to Nieto and Ruiz (2015). The use of EWMA helps the researchers to overcome the problem of volatility clustering. The volatility clustering arises from the fact that asset variances are constantly changing and more specifically big changes in variance tend to cause big changes and vice versa.

2.1 Autoregressive Conditional Heteroskedastic Models

The coping of clustering was the reason why GARCH models were used, with Bollerslev (1986) to be the first to mention them. The simplest form of this model is the GARCH (1,1) that can be expressed using the following equation (3):

$$y_t = \sigma_t \varepsilon_t \text{ i.i.d. (0,1)}$$

$$\sigma_t^2 = \omega + \alpha_1 y_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

where:

- $\sigma_t^2_{t+1}$: is the conditional covariance of the day $t+1$
- $\sigma_t^2_{t}$: is the conditional covariance of the day $t$ (the previous day)
- $\varepsilon_t$: standardized residuals
- $\omega$: intercept coefficient

And with $0 \leq \alpha, 0 \leq \beta, \alpha + \beta < 1$

The ARCH (Autoregressive Conditional Heteroskedastic) models were first introduced by Engle (1982) and later the analysis was continued by Bollerslev (1986). After that, many researches were conducted and therefore there is plenty of literature providing with findings regarding to which method is more accurate in order to estimate VaR. The equation that describes the ARCH models is the following (4)

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots + \alpha_p \varepsilon_{t-p}^2$$

with $0 \leq \alpha, 0 \leq \omega$.
where 
\( \sigma_t^2 \): is the conditional covariance of the day \( t \) 
\( \varepsilon^2 \): standardized residuals of previous days 
\( \omega \): intercept coefficient

Ding et al (1993) suggested that since there is no clear evidence that the conditional variance is a linear function of lagged squared returns, the Asymmetric Power ARCH model could be used in order to estimate VaR accurately and its equation should be the following:

\[
\sigma_t^\delta = \omega + \sum_{i=1}^{q} (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta \alpha_i + \sum_{i=1}^{p} b_i \sigma_{t-i}^\delta \tag{5}
\]

As far as the APARCH models are concerned, Giot and Laurent (2003a,b) on their findings suggest that this model, when it is used with a skewed Student distribution, can calculate VaR in a more accurate way because the exception rates are approximately equal to the expected ones in different confidence levels. This is consistent with the findings of Huang and Lin (2004). To this conclusion, they added that for lower confidence level the normal distribution should be used, while for higher levels the Student distribution is more appropriate. Another research conducted by Degiannakis (2004) concluded that APARCH model and especially the Fractional Integrated APARCH is better when it comes to the calculation of VaR and the tomorrows’ realized volatility. Another important outcome in which Guermat and Harris (2002) and Billio and Pelizzon (2000) have concluded is that when ARCH models are under the Student distribution they tend to overestimate VaR even if the confidence level is 99%. Furthermore Sajjad et al. (2008) suggests that a Markov-switching APARCH model, in which the persistence of volatility can have various values based on whether the volatility is big or not can lead to accurate VaR estimations.
2.2 Generalized Autoregressive Conditional Heteroskedastic Models

In an effort to find the best model, researchers, starting with Bollerslev, have turned to GARCH (Generalized Autoregressive Conditional Heteroskedastic) models. In their work Bali and Theodossiou (2006) by using a combination of the generalized Student distribution with 10 GARCH specifications concluded that TS-GARCH that was first introduced by Taylor (1986) and Schwert (1989), and the EGARCH introduced by Nelson (1991) were the most accurate models to calculate VaR. More specifically the Exponentially GARCH model was established in order to deal with the weakness of GARCH to take into account the asymmetry of the financial data and it is expressed by the following equation where the $\gamma_i$ parameters account the asymmetry:

$$\log (\sigma^2_t) = \alpha_0 + \sum_{i=1}^{q} \left( \alpha_i \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \gamma_i \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right) + \sum_{i=1}^{p} \left( b_i \log (\sigma^2_{t-1}) \right)$$  \hspace{1cm} (6)

The conclusion is that models that do not take into account the asymmetries, are not sufficient and tend to underestimate the VaR, as has been supported by Brooks and Persand (2003a) and produce less accurate forecasts (Angelidis et al. 2004).

The superiority of GARCH model was presented by Guermat and Harris (2002) who concluded, after having backtested their findings, that the GARCH models are better than the rest not only under the normal but also under the Student distribution. Furthermore Boucher et al. (2014) has reached to the decision that when it comes to the estimation of VaR Risk metrics and GARCH, that the parametric models, are among those that provide the most accurate results. Nieto and Ruiz (2010) conclude that the forecasts of VaR based on conditional variance using GARCH type models as well as symmetric leptokurtic errors have adequate results. On the contrary, Koopman et al. (2005) suggest ARFIMAX, the fractionally integrated auto regressive moving average with exogenous variables, as a better model than GARCH and this conclusion was reached after a close research on VaR estimates of the S&P100. The effectiveness of GARCH models was pointed out also by Giot (2005). When it comes to the intraday horizon, he underlined that GARCH model under the Student distribution provided the most accurate VaR forecasts. In addition it is pointed out by his work that once the
volatility in an intraday basis was accounted, the results between the daily and intra-day VaR were similar.

In a more general basis Angelidis et al.(2004) indicates that despite the confidence levels and the distributions, the more flexible a GARCH model is, the better forecasts can provide regarding the VaR.

On the contrary, one of the main drawbacks of this category is the assumption of normality since it is inconsistent with the reality; risk factors do not follow the normal distribution and thus in general the results based on this category underestimate the VaR.

3. Non parametric models

3.1 Historical Simulation

Historical simulation (HS) is a quite popular method among the financial institutions as Perignon and Smith (2006) point out for calculating there can be found VaR. What’s more Pérignon and Smith (2010b) stress out that almost 75% of the banks estimate their VaR by using this method. It applies current weights to a time series data of historical index returns and it consider that every day a profit or a loss is accounted. These profits and losses are then formed in a distribution; the percentile of this distribution is calculated according to the desired confidence level and then are rearranged and sorted in ascending order. One of the characteristic of this method is that it doesn’t underlie any assumption for normality regarding the distribution, the VaR is calculated using the actual price movements and it can capture the non normalities of market return as Bangia, Diebold & Schuermann (1999) underlie. This is one of the main advantages of this non parametric method is that it doesn’t make the assumption that the risk factors follow the normal distribution; this is a positive aspect since it provides the researchers with the ability to account for fat tails and skewness.

The equation (7), that is presented below describes the HS:

\[ VaR_t = F^{-1}_\alpha \{ (y_{i})^{\tau-1}_{t-\tau} \} \]  

(7)
where

\( F_{\alpha}^{-1} \) is the \( \alpha \)th quantile function of the assumed distribution

Apart from that, another characteristic of the HS approach is the assumption that the distribution which is derived by past data can explain the distribution of the future returns.

According to literature, one of the major drawback of this so usual method is it’s assumption that the returns are independent and identically distributed (i.i.d). HS places equal weight to the daily returns without taking into account the timing of the returns, that is recent and past returns are accounted the same when the VaR is calculated. Bollerslev (1986) points out that in periods of high or low volatility the clustering effect is present. This is not the case when the returns are not assumed i.i.d and it turns out to be more precise since the recent returns tend to depict better the volatility than older ones. As a result, Zikovic and Aktan (2011) introduced the weighted historical simulation (WHS) that place larger weights to the most recent observations. Another paper by Boudoukh, Richardson, and Whitelaw (1998) suggest a hybrid approach as an improvement of the HS since it generalize the HS so as to place more weight to the most recent returns. This is achieved by placing weights, the sum of which was equal to unity, but they were decaying exponentially. Once the weights are accounted the VaR is measured using the empirical CDF of returns. The idea of the BRW model is that it could be helpful especially during periods with high volatility for example in time of a crash since it provides better estimations regarding VaR. But Pritsker (2001) finds that both BRW as well as the HS pay attention only on the lower tail of the distribution and thus assume that if a change on the upper tail of the distribution takes place, it will not affect the lower tail. According to Pritsker (2001), the fact that either BRW or HS do not take into account changes in VaR is linked to the extent that VaR is possible to change over a time horizon without being detected. As it is shown, there is a 31%probability that increases in VaR will not be depicted in the model. A solution to this phenomenon would be the use of GARCH (1,1).

As empirical analysis reveals the correlation between the VaR estimates deriving from both the BRW and HS and the true VaR are quite high meaning that the methods tend to move together in the long run. The problem that is raised though is that the above
methods do not capture in time the changes in volatility and therefore the VaR estimates are not that accurate.

What’s more the conditional variances of the returns are not calculated by the Historical Simulation model. In an effort to deal with this limitation Hull and White (1998) suggest that the HS should be applicable by taking into consideration the volatility estimated with a use of a GARCH model.

After having presented the main characteristics of the models, it is constructive to compare them with other methods used like the variance–covariance method with equally weighted observations and variance covariance with exponentially declining weights as Pritsker (2001) has presented.

As far as the V-C with equally weighted observation, the conclusion is that it is very close to the models under scrutiny since they all share the same characteristic; that is they do not reckon time possible changes in volatility.

When the BRW and HS are compared with the V-C with exponentially declining weights the results differ since the V-C model tend to outperform the rest. The main reason for this is that the C-V model takes into account the changes in volatility regardless of whether there are profits or losses on the portfolio while the HS and BRW consider the changes only when losses take place. What is more, Pritsker (2001) points out that the exponentially weighting procedure share characteristics with the GARCH (1,1) model that is more accurate in forecasting VaR and therefore is more sensitive in changes in conditional volatility.

4. Semi parametric models

Although both parametric and non parametric models are quite spread when it comes to the VaR forecasting, the inefficiencies that exist have made it necessary to introduce more accurate models in order to capture market risk and VaR. A new category of models are the semi parametric such as the Filtered Historical that was first presented by Hull and White (1998) and Barone-Adesi et al. (1999) in an effort to preserve the positive aspects of both Historical Simulation and Conditional volatility models and to create a new approach. This effort has established the Filtered Historical Simulation (FHS) which is one of the most common semi parametric model.
4.1 Filtered Historical Simulation

FHS is a method that estimates the quantiles based on the Historical Simulation while it accounts the skewness and the kurtosis based on the use of GARCH methods, as O’Brien and Szerszen (2014) mention and calculates the variance by using a parametric volatility model; so as to combine the conditional heteroskedasticity and non-normality of the risk factors (Pritsker 2001).

This is achieved by bootstrapping returns within a conditional volatility model such as GARCH.

This method is considered to provide better forecasts as Barone-Adesi and Giannopoulos (1999) point out since it depicts the recent data of the market. Also the FHS provide the percentiles in the tails of the distribution and this is a major improvement compared to the HS. The aforementioned characteristic of FHS not only facilitates the researcher to account for the tails of the distribution since the data that must be collected regard a shorter period but also provide him with the opportunity to stress test since it accounts the whole distribution.

The above conclusion, regarding to the superiority of the FHS, has also been underlined by Angelidis and Benos (2006) who suggest that at higher confidence levels, FHS has more accurate results. In addition one of the main advantages of FHS is that takes into consideration the skewness the fat tails as well as the clustering effect.

The equation that best describes this semi parametric method is the following:

\[ V\alpha R_{t+1|t} = F_\alpha \left( \left( z_{t+1-i|t} \right)^T \theta \right) \sigma_{t+1|t} \]  

(8)

where:

\[ z_{t-i} = \frac{\epsilon_{t-i}}{\sigma_{t-i}} \]  
and they are the standardized residuals

The advantage of the FHS compared to the remaining models is also described by Angelidis et al. (2006) who conclude that when the FHS is combined with an ARCH or
a GARCH volatility model, then the results are more accurate compared with the other methods used to forecast VaR since it depict faster changes in the market.

According to the literature there are two ways in order to present the returns. There is the devolatilising procedure according to which the returns are divided by an estimate of the volatility the day the return took place and the second is the revolatilising in which the returns are multiplied with an estimate of the day that VaR was measured. These procedures present how the filtering can affect aspects of the distribution like skewness and kurtosis. One of the characteristic of FHS concerning the margins is that when the volatility is high the margin will be higher and vice versa. Gurrola-Perez& Murphy (2015) and this happens since this method gives emphasis on the current condition of the market. That is the reason why the Filtered Historical Simulation outperforms the Historical Simulation at any confidence level.

4.2 Extreme Value Theory

Extreme Value Theory is another semi parametric model that is commonly used in order to calculate VaR. According to Jorion (2007) it is a branch of statistics tailor that has been introduced in order to overcome the difficulties arising by the extreme events. Jorion (2007) points out that EVT is applicable only on the tails of a distribution since it estimates the extreme events making this property the main advantage of this method. One of the characteristics of the EVT is that is not difficult to be implemented since a Student distribution with 4 to 6 degrees of freedom can provide all the necessary elements and lead to accurate forecasts. Chan and Gray (2006) inferred the above after having tested EVT with other famous both parametric and non parametric models. The EVT is expressed using the following equation (9).

\[ V_{\alpha} R_{t+1|t} = \sigma_{t+1|t} \left( \frac{\alpha}{T_{u}/T} \right)^{-1/\tau} \]  \hspace{1cm} (9)

where

\( \alpha \): confidence level

\( \tau \): Hill estimator of the tail index
**T_u**: number of observations beyond the threshold u which is set to be 5% of the total sample size T

Silva and Mendes (2003) proposed that the EVT is more accurate in forecasting VaR; this conclusion was reached after having analyzed ten market indices in Asia. The returns were not under the normal distribution and therefore the tails could be accounted with the use of EVT. Additionally in the papers of Cencay et. Al (2003) and Cencay and Selcuk (2004) it is proved that EVT performs better that the rest of the parametric and non parametric models at bigger quantiles, after having studied the VaR of nine stock markets of emerging countries. Likewise Bekiros and Georgoutsos (2005) suggest that EVT compared the predictive ability of various methods and have concluded that methods related to EVT like Peaks Over Threshold (OPT) as well as Blocks Maxima (BM) offer better forecasts. OPT is a method in which all the observations that exceed a given high threshold (u) are modeled separately, while BM is a method that divides the sample into m subsamples that contain n observations and takes the maximum of each subsample as Nieto and Ruiz (2015) explain. Another study that could be used additionally to the previous ones is that of Jesus et al (2013). This paper has focused on the risk of foreign exchange between Dollar and Peso and compared the results arising from both historical simulation and EVT. Once again the findings supported the opinion that VaR under the EVT methodology is better estimated.

But as all methods, EVT has some drawbacks as well. The asymptotic characteristics of the EVT lie on the fact that the returns are i.i.d which is usually not the case. In an effort to deal with this negative aspect many researchers have made propositions like the SEMPP, the Self Exciting Market Point Processes, since they could account the time between exceptions, Chavez-Demoulin, Davidson, and McNeil (2004) introduced the Hawkes POT model while Herrera and Schipp (2013) suggested the ACD POT model.

Overall, according to the literature presented the EVT is one of the most accurate methods of estimating VaR. However there are studies like Sener et al. (2012) who point out that VaR under EGARCH models tend to have better performance.
5. Backtesting

All the models that are being used in the literature in order to be considered trustworthy undergo checks that are known as model validation. There are many types of procedures like stress testing as well as backtesting. In this study emphasis will be given on the backtesting. According to Jorion (2007) “Backtesting is a formal statistical framework that consists of verifying that actual losses are in line with projected losses and it involves the systematic comparison of the history of VaR forecasts with their associated portfolio returns”. If the models proved to be inefficient then they must be reviewed. The importance of the backtesting can be realized since the Basel Committee promotes this procedure as a model validation to all financial institutions. In order for the procedure to fulfill the Basel requirements the backtesting should be based on at least 250 one-step-ahead VaR forecasts. A model is considered to be adequate when the number of exceptions when estimating VaR is in line with the confidence level, since if the number of observations is either more or less than the confidence level, the risk is not correctly estimated. A VaR forecast is effective if it is conditionally unbiased (Nieto and Ruiz 2015), that is if the following equation holds:

\[ F_{t-1}[\frac{l_{t}}{\sigma}] = \alpha \quad (10) \]

There are many techniques that are being used by the researchers, but those proposed by Kupiec (1995) and Christoffersen (1998, 2003) are among the most widely used and therefore this study will focus mainly on these two methods. The benefits of the Backtesting procedure according to literature are plenty since the related parties can determine which model is effective in order to calculate VaR. As a result, when the VaR forecasts are accurate, the requirements of the Basel Committee for the Banking Supervision are fulfilled.

5.1 Unconditional coverage

One of the methods that will be presented will be the one introduced by Kupiec (1995). According to this test the number of the times that VaR cannot predict the realized losses follow a binomial distribution. The null hypothesis which states that the realized
losses are equal to the calculated ones, \( H_0 : \text{E}[\Gamma_t^a] = \alpha \), should not be rejected. The equation that depicts this criterion is the following

\[
LR_{UC} = 2 \ln \left[ 1 - \left( N_t \frac{T-N}{T} \left( \frac{N_t}{T} \right)^N \right) \right] - 2 \ln \left[ (1 - \rho)^{(T-N)} \rho^N \right] \sim X^2
\]  

(11)

Where \( N \) is the proportion of failures, that is, the days in which the losses were higher than the VaR forecasts and follows a \( \chi^2(1) \) distribution.

In addition as far as the unconditional coverage criterion is concerned, Escanciano & Pei (2012) point out that it is not consistent when it comes to detect the non optimal VaR forecasts based on HS and FHS methods. The weaknesses of the unconditional coverage have been also presented by de la Pena et al. (2007). In this paper the author proposes a switch between the null and alternative hypothesis in order to increase the odds of selecting the right model.

### 5.2 Independence coverage

Another notable backtesting criterion that is commonly presented in the literature is the one proposed by Christoffersen (1998,2003). It is a more sophisticated criterion since it measures if the total number of failures is statistically equal to with the expected ones. Apart from that it examines if the VaR violations are independent. In order to examine the first assumption the Kupie’s (1995) equation should be used, while the second is calculated by a likelihood ratio statistic \( LR_{in} \) using the following equation:

\[
LR_{IND} = 2 \ln \left( \frac{(1-\pi_0^1)^{n^00} \pi_{01}^{n^01} (1-\pi_{11})^{n^10} \pi_{11}^{n^11} - \ln((1-\pi_0)^{n^00+n^10} \pi_0^{n^01+n^11}))}{(1-\pi_0)^{n^00+n^10} \pi_0^{n^01+n^11}} \right) \sim X^2
\]  

(12)

Where the null hypothesis is \( H_0: \text{E}[\Gamma_t^a \mid \Gamma_{t-1}^a] = \alpha \).

### 5.3 Conditional coverage

The conditional coverage is a combination of the above backtesting criteria and the likelihood ratio statistic is the following

\[
LR_{CC} = LR_{IND} + LR_{UC}
\]  

(13)
Each statistic follows the $\chi^2(1)$ distribution while their combination (CC) follows a $\chi^2(2)$.

The aforementioned tests help the researchers determine whether the model is accurate or not, but this procedure is not always easy since many methods at the end are not characterized as trustworthy (Angelidis and Degiannakis 2007). Backtesting is a difficult procedure and therefore many were the researchers who tried to implement improvements by introducing new models. One of them is the procedure suggested by Lopez (1999) which is considered to be the second stage, after the implementation of the two criteria. Lopez suggested a new method that would base the VaR evaluation on a loss function. The idea behind this model is to provide a close to reality utility function that would take into account more than one volatility forecasting technique. For this purpose the distance between the failures, if a violation happened, should be measured. The model is described by the following equation (14).

$$\Psi_{t+1} = \begin{cases} (y_{t+1} - \text{VaR}_{t+1|t})^2 & \text{if a violation takes place} \\ 0 & \text{otherwise} \end{cases}$$

(14)

where:

$\Psi$: the total loss value

The VaR model that produces the most accurate forecasts would be the one that minimizes the total loss value.

Although the backtesting is a helpful tool in order to find the most appropriate and effective model, Escanciano and Olmo (2010), stress out that both conditional and independence procedure can provide with wrong results since they do not take into consideration the parametric estimation. In order to overcome this weakness they proposed a dynamic parametric VaR.

A very interesting outcome presented by Alexander and Sheedy (2008) after having backtested the Historical Simulation, one of the most commonly used methods for estimating VaR, is that it is not that effective in many cases and therefore it is not suitable when it comes to stress testing.
6. Expected Shortfall

VaR is considered to be an accurate method for measuring the market risk and the Basel Committee urge all he financial institutions to calculate VaR. However, it still tends to have some weaknesses. Artzner et al (1997, 1999) concludes that in a portfolio, the overall VaR estimation could be misleading, since computing the individual VaRs and then adding them up may lead to underestimations. In the same direction Angelidis and Skiadopoulos (2008) point out that VaR is not a coherent measure of risk. A measure of risk should fulfill four properties in order to be considered coherent, that is to have sub-additivity, homogeneity, monotonicity and risk free condition. (Angelidis and Degiannakis 2007).

The limitations of VaR have been lined up by O’Brien and Szerszen (2014) since it cannot provide with information in the case that the potential loss is bigger that the forecasted VaR and account for tail risk.

In order to overcome the weaknesses of VaR, Expected Shortfall (ES) was introduced. ES is a more conservative measure of risk as it pays more attention on the less profitable outcomes and it is defined as the average loss over the losses that have exceeded VaR and its equation is depicted below:

\[ ES(a) = E[y_x | y_x \leq -VaR(a)] \tag{15} \]

Furthermore, Basak and Shapiro (2001) proposed the limited expected losses-based risk management (LEL-RM), a model that put more weight on the expected losses, once they take place.

ES provide information regarding a possible loss, bigger than the forecasted VaR, making it an accurate measure of risk. In most cases the ES is calculated using the past Profits and Losses under a GARCH model. The study of O’Brien and Szerszen (2014) reveals that during pre crisis periods the estimates are quite accurate, although this is not the case in periods of high volatility where they tend to be understated.

On the other hand Expected Shortfall do not come without a cost. Yamai and Yoshiha (2005) cite that in order for the ES to provide the same accurate forecasts as the VaR,
the use of a more data and a bigger sample is required. Last but not least according to their study in cases of a heavy tailed distribution, the ES has low accuracy.

7. Empirical Investigation and Methodology

The purpose of this study is to evaluate the predictive ability of VaR methods regarding the European Stock Exchanges the years that followed the burst of the global economic crisis of 2008. In order to do so, the dissertation focuses on the closing prices of six European countries that were split into two groups. The first group consists of the three largest European economies; Germany, France and United Kingdom. The second group contains European countries that have been under EU-IMF rescue programs and more specifically Iceland, Portugal and Ireland. The countries that were under a rescue program faced a number of changes in their economies. When the global financial crisis burst, their market volatility increased dramatically and so were the spreads, leading to an increase in the borrowing rates. As a result, the countries’ bonds have stopped being traded in the financial markets since the countries were borrowing money from the parties involved. On the other hand the countries of the first Group, that is the richest countries in Europe, although their economies were influenced by the global financial crisis, their economies and their stock indices didn’t undergo such a dramatic change.

This dissertation will try to investigate whether the severe conditions that some European countries faced during the late financial crisis as well as their participation in a program lead to differences between the two Groups when it comes to the VaR calculation.

Germany is the richest country in the Europe and despite the latest financial crisis the country’s GDP was increased 1,7% during 2015. Similarly, French GDP was higher in 2015 by approximately 1,2% compared to the previous year, making France the third bigger economy of the EU. Last but not least, the UK is the second larger economy in the Europe and has been the fastest growing economy of G7 countries for four years in a row.

On the contrary countries like Portugal, Iceland and Ireland have seen their economies shrinking and facing even the danger of default after the burst of the 2008 financial crisis. The results of the crisis in Portugal started in 2010. The political instability and
the increase of interest rates and CDS urged Portugal to turn to the EU and the IMF for a rescue package of 78€ billion in order to stabilize the economy. The country exited the €78 billion program in November 2014.

Ireland at the end of 2007 was considered to be the 4th richest country based on the GDP per capita, but the collapse of Lehman Brothers and the extended financial exposure that the country had, lead Iceland to one of the worst economic crisis of the country’s modern history. The nationalization of Banks increased the country’s debt dramatically while the capitalization of the Icelandic stock exchange was decreased by 90%. On November 2008 the country agreed with the IMF for a package of 1,5€ billion and made a 2,5€ package agreement with Norway, Sweden Denmark and Finland. On 31st of August 2011 the program was successfully completed.

Ireland was another European country that faced the danger of collapse and turned to a bailout program in order to avoid bankruptcy. According to the Central Statistic Office (CSO), Ireland was the first member of the euro zone that faced recession. On November 28 2010, Ireland made a 85€ billion agreement with EFSF and the IMF in order to deal with the crisis. On December 2013 Ireland exited the bailout program and managed to have access to the financial markets.

The purpose of the study is to shed light on whether the performance of the countries’ economies the last years affects the predictability of the VaR methods. The sample size is a very important factor and in the literature, there can be found many papers that try to conclude which sample size is more appropriate. There are those in favor of big samples such as Hendricks (1996), Vlaar (2000) and Danielsson (2002) who suggest that when the sample size is bigger, the VaR estimations will reflect the reality better. On the other hand, Hope (1998) points out that choosing smaller samples is better since the estimation based on smaller samples is more accurate. Frey and Michaud (1997) seem to agree with this opinion since according to their paper small samples depict better the changes that might occur.

The horizon under scrutiny in this dissertation ranges between 1st January 2000 and 29th July 2016 for the five out of six countries. Regarding Iceland the data is between 1st January 2001 and 29th July 2016. The data set was divided into two periods. The first, between 1st January 2000 to 31st December 2007, was used for the in-sample analysis. The second starting from January 1, 2008 and ending in July 29 2016 was used in order
to forecast the VaR methodologies. The year 2008 was set as the starting point of the out of sample analysis, since then the burst of the latest economic crisis took place. The non trading days were excluded from the data as well as the missing values and missing-zero values in order to add more accuracy, the daily returns were collected and used since they can capture better the conditions on the European Stock Exchanges. Furthermore by using daily returns the phenomenon of noise and excessive variance could be reduced. In addition, the adjusted closing prices were used, since they provide a better insight regarding the corporate actions that took place before the opening of the Stock Exchange. Apart from that the models were estimated with log returns since their use offer benefits to the researchers, such as numerical stability since it deals with the arithmetic underflow as well as time additivity.

8. Data Analysis

8.1 Descriptive statistics

The analysis begins with the Descriptive Statistic which is a useful tool in the estimation of VaR since it is calculated by taking into consideration the standard deviation as well as the mean of the countries’ indices. Table 1 summarizes all the relevant information regarding the descriptive statistics for all the countries that are examined. The mean, that corresponds to the average return that the European Stocks had, for all of the six countries is close to zero meaning that there is no predictability in the market; there are no persistent long term positive or negative returns. In the case of France, UK and Ireland the mean is negative. Iceland appears to have the biggest return approximately 0.089%, while the standard deviation provides info regarding their volatility in the time horizon examined.

The analysis also reveals that the returns of the Indexes are not following the normal distribution, since the skewness is different than zero and the kurtosis is bigger than three. What’s more the Jarque–Bera test confirms the non normality of the distributions. According to the test the rejection of the null hypothesis ($H_0$) indicate non normality, while the rejection of the alternative one ($H_1$) indicates normality. As it can be seen, the p value is smaller than the critical value of 0.05 meaning that the $H_0$ hypothesis is rejected in all countries.
Table 1: Descriptive Statistics of the daily log returns

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>France</th>
<th>UK</th>
<th>Iceland</th>
<th>Ireland</th>
<th>Portugal</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.0000877</td>
<td>-0.0000258</td>
<td>-0.0000239</td>
<td>0.0008900</td>
<td>-0.0002410</td>
<td>0.0000466</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.0007940</td>
<td>0.0003370</td>
<td>0.0004120</td>
<td>0.0011900</td>
<td>0.0009690</td>
<td>0.0002910</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>0.0755270</td>
<td>0.0700230</td>
<td>0.0590260</td>
<td>0.0504360</td>
<td>0.0354050</td>
<td>0.0429760</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>-0.0665220</td>
<td>-0.0767810</td>
<td>-0.0588530</td>
<td>-0.0447800</td>
<td>-0.0590600</td>
<td>-0.0463170</td>
</tr>
<tr>
<td><strong>Std Dev</strong></td>
<td>0.0154920</td>
<td>0.0139680</td>
<td>0.0113660</td>
<td>0.0086620</td>
<td>0.0118000</td>
<td>0.0092800</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0.0457990</td>
<td>-0.0927390</td>
<td>-0.2227210</td>
<td>-0.5286370</td>
<td>0.6321350</td>
<td>-0.4202930</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>5.7529040</td>
<td>5.9616490</td>
<td>6.0733020</td>
<td>6.6159080</td>
<td>5.1792240</td>
<td>5.9339030</td>
</tr>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>642.3550</td>
<td>748.1232</td>
<td>812.8755</td>
<td>1021.2770</td>
<td>532.1234</td>
<td>782.7954</td>
</tr>
<tr>
<td><strong>Probability</strong></td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The Figure 1 below present the distributions of all countries.

![Figure 1: Frequency Distribution](image1)

In order to provide an insight into the stock Indexes returns, the study provides also a plot of the Returns for the years under scrutiny. As it is presented in the below Figures there seem to be a pattern regarding the Returns. Periods characterized by high volatility are to be followed by periods of high volatility as well. Similarly periods of low
Volatility are followed by periods that tend to have low volatility. In addition an interesting finding is that the returns are continuously compounded. This pattern is visible on the Figures presented for all the countries. One interesting point is presented in the plot graph of Iceland where on October 2008 there has been a significant decrease. On October 13th 2008, there was observed a large scale decrease in the index return from a 2771.16 to a 919.25 because Iceland’s central bank has abandoned an attempt to peg its currency with the euro and dollar. On Figure 2 the plot of returns can be seen.

Another important matter for which the data were tested is the whether they are stationary or they have unit root. The meaning of stationarity is quite important when it comes to the time series data since it can affect the behavior of the data. If the data are non stationary and a for example a shock occurred, its persistence would last forever. In order to make sure whether the data are stationary the Dickey Fuller test was used, according to which the null hypothesis ($H_0$) indicates unit roots while the alternative
(H\textsubscript{1}) indicates stationarity. Figure 3 below depicts the results according to which the time series for all the six countries are stationary since the null hypothesis is rejected (p<0.05).

<table>
<thead>
<tr>
<th>Country</th>
<th>Augmented Dickey Fuller Unit root test</th>
<th>Null Hypothesis: there is unit root</th>
<th>Exogenous: Contstant</th>
<th>Lag Length: 2 (Automatic - based on SIC, maxlag= 25)</th>
<th>t- statistic</th>
<th>Prob*</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>Augmented Dickey Fuller test stat</td>
<td>-29.5810</td>
<td>0.0000</td>
<td>* MacKinnon (1996) one sided p- values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>Augmented Dickey Fuller test stat</td>
<td>-23.8755</td>
<td>0.0000</td>
<td>* MacKinnon (1996) one sided p- values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>Augmented Dickey Fuller test stat</td>
<td>-45.9201</td>
<td>0.0001</td>
<td>* MacKinnon (1996) one sided p- values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>Augmented Dickey Fuller test stat</td>
<td>-46.7089</td>
<td>0.0001</td>
<td>* MacKinnon (1996) one sided p- values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td>Augmented Dickey Fuller test stat</td>
<td>-41.4613</td>
<td>0.0000</td>
<td>* MacKinnon (1996) one sided p- values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iceland</td>
<td>Augmented Dickey Fuller test stat</td>
<td>-42.4333</td>
<td>0.0000</td>
<td>* MacKinnon (1996) one sided p- values</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3: Dickey-Fuller test results
The first method presented for calculating VaR in this dissertation is the Variance-Covariance, since it is one of the simplest one. This method takes into account the closing prices of the Stock Exchanges and based on the probability theory calculates the maximum loss that the Index could face within a day. It is named Variance Covariance method since it uses the standard deviation of the closing prices and assuming that the normal distribution is followed, the VaR is calculated using a certain confidence level. Table 2 presents the VaR for the countries under scrutiny.

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>France</th>
<th>UK</th>
<th>Portugal</th>
<th>Ireland</th>
<th>Iceland</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR</td>
<td>-2,54%</td>
<td>-2,48%</td>
<td>-2,02%</td>
<td>-2,04%</td>
<td>-2,33%</td>
<td>-3,37%</td>
</tr>
</tbody>
</table>

Table 2: VaR results under the Variance Covariance method

As far as the VaR is concerned, it was calculated by taking into account the mean and the variance of each country while the percentile that was used is 1,645 that correspond to a 95% confidence interval. The VaR results represent the maximum loss that is expected to take place in the countries under scrutiny based on the confidence level and time horizon presented above. It can be deduced that as a percentage it is close to all examined countries irrespective of whether the country belongs to the first or the second Group. The only divergence that can be observed regards Iceland indicating that it the maximum expected loss is the bigger among the scrutinized countries.

Overall, although the Variance Covariance method is easy to implement it is not the appropriate method to forecast VaR. This was concluded after having applied the backtesting procedure by using the three main criteria found in the literature. The first is the one proposed by Kiupiec and is known as unconditional coverage. In addition there is the one proposed by Cristoffersen and it is called independence coverage. Last but not least the combination of both is the conditional coverage. The conclusions derived are the following; regarding the three biggest economies (Germany, France and UK) the method is rejected at all significance levels by using all the three criteria. As far as the second Group is concerned the V-C method is also rejected in Portugal, and Iceland, with the only exception to be the case of Ireland where it is rejected at a 99% confidence level but not at the 95%. This outcome reveals that V-C method could not be used accurately in order to forecast VaR for the European Stock Exchanges during the latest financial crisis. In addition, the participation of a country to a rescue package does
not seem to affect the predictive ability of VaR. Since many times is the above method underestimates the true VaR, more methods should be implemented in order to make accurate estimations.

### 8.2 Historical Simulation

Historical Simulation is also a commonly used method for forecasting VaR, since it is easy to implement. The results presented below tend to differ from those of the V-C method, since there are differences in the HS VaR forecast among countries in the same Group. Specifically UK’s HS VaR is significantly lower than those of Germany and France which are identical. The same applies to the countries of the second Group. Portugal’s HS VaR is lower compared to VaR of Ireland and Iceland.

As far as the examined European Stock Exchanges are concerned, the method seems to be inadequate when it comes to the forecasting of VaR. More specifically, regarding 95% confidence level in German, French and UK index the method is rejected by all the three criteria. At a 99% confidence level it is not rejected for the whole Group under all the criteria. The results for the countries of the second Group indicate that the method is rejected at all confidence levels. That means that the method cannot provide accurate forecasts in countries that have faced severe economic problems during the crisis. The following Table 3 summarizes the VaR results for all the countries. The positive aspect of the HS VaR method is that it doesn’t take for granted the normality of the returns distributions; this was presented above in the descriptive statistics since none country follows the normal distribution. Therefore fat tails and skewness can be accounted.

The Figure 4 below depicts the HS VaR at a significance level of 5% for the examined Group of the countries analyzed above, while the Tables provide the graphs for the two Groups. The analysis reveals that the returns of the Indexes are not following the normal distribution, as the V-C method assumes, since the skewness is not different than zero and the kurtosis is not equal to three. The HS VaR for the countries represents the maximum loss that can occur at a 95% confidence level and at the specified time horizon. In addition Table 3 depicts the ongoing HS VaR for the time under scrutiny.
Apart from that, the below Figure 4 depicts the graphical presentation of the HS VaR.

![Figure 4: HS VaR graph](image)

### 8.3 Filtered Historical Simulation

In an effort to overcome the disadvantages of the Historical Simulation, the researchers have turned to Filtered Historical Simulation which combines the positive aspects of both parametric and non-parametric models. Since it uses a combination of econometric models and historical returns, the risk forecasts are derived from the tails of the distribution.
FHS uses a combination of nonlinear econometric models and past returns to build the probability distribution of possible values that the asset (risk factor) could take in the days ahead. Risk estimates are directly derived from the tails of the distribution. One of the major advantages of this method is that it overcomes to a great extent the biasness that is easy to appear when it comes to the use of historical data.

The FHS was used in order to make forecasts regarding VaR in the years that followed the burst of the global financial crisis and that is why an out of sample analysis was undergone. Graph 5 depicts the FHS VaR.

![Graph 5: FHS VaR graph](image)

### 8.4 Expected Shortfall

The Expected Shortfall was introduced as a new approach for calculating VaR in order to overcome the inefficiencies of the VaR methods. ES provides the probability under a certain confidence level that the loss will be greater than that of VaR. It is a more conservative way of calculating market risk since the less profitable outcomes are given more weight. The Expected Shortfall is calculated by the weighted average of the VaR, as well as the losses that are bigger than VaR. The literature review shows that during
the last few years, ES was increased in popularity since the method satisfies the sub-additivity property as Chen (2008) suggests.

Table 4 below summarizes the results for the scrutinized countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Expected Shortfall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>-4.61%</td>
</tr>
<tr>
<td>France</td>
<td>-4.49%</td>
</tr>
<tr>
<td>UK</td>
<td>-3.76%</td>
</tr>
<tr>
<td>Portugal</td>
<td>-3.82%</td>
</tr>
<tr>
<td>Ireland</td>
<td>-4.67%</td>
</tr>
<tr>
<td>Iceland</td>
<td>-4.89%</td>
</tr>
</tbody>
</table>

Table 4: Expected Shortfall

The main conclusion is that Expected Shortfall does calculate the expected loss in a more conservative way. The results are significantly higher compared to the VaR methods presented above. It means that according to this method the loss that the European Stock Exchanges could face exceeds the loss forecasted by VaR. The only outcome that remains common when compared with the HS VaR is that both the UK from the Group of the richest countries and Portugal from the second Group continue to have a lower ES percentage as compared to the other countries of their Groups.

8.5 EVIEWS results

The econometric package that is used in this dissertation is Eviews, which is widely used in research especially in the fields of economics. In the following sections the results of the econometric analysis are presented.

8.5.1 ARCH

ARCH is a non linear model that is commonly used in finance when it comes to time series data. The main advantage of the ARCH models is that they do not assume that the variance is constant and therefore they can describe how the variance of the errors evolves through time (Brooks, 2008). In addition, the ARCH models can depict
volatility pooling that is the tendency of large changes to be followed by equally large ones. In all the models presented in this dissertation the lag length is 1 and the Autoregressive model was a first model AR(1). More specifically, below the ARCH(1) with AR(1) was used.

The p-values of the ARCH models can make researchers decide whether to reject the null hypothesis that assumes that the coefficient values are zero or not. The F statistic will be used in order to test whether the $H_0$ is rejected or not. If the p-value is smaller than the critical one, then the null hypothesis is rejected. More specifically the hypothesis for the ARCH models is presented below.

$H_0$: The coefficients $\alpha$ are zero (homoskedasticity)

$H_1$: The coefficients are different from zero (heteroskedasticity)

In the Table 5 that follow the results for the countries are presented.

| Table 5: ARCH(1) with AR(1) results |

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000637</td>
<td>0.000242</td>
<td>2.631536</td>
<td>0.0095</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.031603</td>
<td>0.024513</td>
<td>-1.287397</td>
<td>0.1945</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESI(1)^2</td>
</tr>
<tr>
<td>QARCH(1)</td>
</tr>
</tbody>
</table>

R-squared: 1.97E-06  4.83E-07  4.074768  0.0000  0.0000  0.0000  0.0000

Adjusted R-squared: -0.000503  S.D. dependent var  0.015650

S.E. of regression  0.155503  Akaike info criterion  -5.94506

Sum squared resid  0.486443  Schwartz criterion  -6.989052

Log likelihood  989586  Hannan-Quinn criter.  -6.99494

Durbin-Watson stat  2.007826

Inverted AR Roots  -0.03

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000148</td>
<td>3.83E-06</td>
<td>38.69923</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESI(1)^2</td>
<td>0.250811</td>
<td>0.029774</td>
<td>8.437400</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 0.000303  Mean dependent var  7.78E-06

Adjusted R-squared: -0.001957  S.D. dependent var  0.015968

S.E. of regression  0.013955  Akaike info criterion  -5.76118

Sum squared resid  0.394410  Schwartz criterion  -5.75070

Log likelihood  5840.956  Hannan-Quinn criter.  -5.757094

Durbin-Watson stat  2.016421

Inverted AR Roots  -0.01

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.004121</td>
<td>0.000299</td>
<td>13.79623</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESI(1)^2</td>
<td>0.696195</td>
<td>0.007740</td>
<td>86.00758</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 3.47E-05  1.45E-06  23.94459  0.0000

Adjusted R-squared: -0.000854  S.D. dependent var  0.000186

S.E. of regression  0.038645  Akaike info criterion  -5.40874

Sum squared resid  2.806633  Schwartz criterion  -5.43866

Log likelihood  5525.608  Hannan-Quinn criter.  -5.44682

Durbin-Watson stat  2.805562

Inverted AR Roots  -0.06
According to the ARCH model above, at a 5% confidence level, the coefficient’s Resid(-1)^2 p-value is lower than the critical one for all countries indicating that the $H_0$ is rejected and therefore there is heteroskedasticity. Since the $H_1$ cannot be rejected it means that at least one of the coefficients is significant. Therefore a GARCH model should be implemented.

### 8.5.2 GARCH

The GARCH model that was first introduced by Bollerslev (1986), allows the conditional variance to depend on its previous own lags. The positive aspect of GARCH models is that they are more parsimonious and avoid over fitting. This section describes the results for GARCH (1,1) model that can be seen in the following tables.

The hypotheses for the models are the following:

- $H_0$: The coefficients are not statistically significant
- $H_1$: The coefficients are statistically significant
**Table 6: GARCH(1,1) results**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.090863</td>
<td>0.001242</td>
<td>2.35136</td>
<td>0.0885</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.031803</td>
<td>0.024513</td>
<td>-1.297379</td>
<td>0.1946</td>
</tr>
</tbody>
</table>

**Variance equation**

- R-squared: 0.090863
- Adjusted R-squared: 0.090863
- S.E. of regression: 0.001242
- Sum squared resid: 0.001242
- Log likelihood: 577.5295
- Hansen-Quinn criter.: -3.89924
- Durbin-Watson stat: 2.07626

Inverted AR Roots: 0.9

**Dependent Variable: OGERMANY**
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 2009/3/5 Time: 12:11
Sample (adjusted): 2007
Included observations: 206 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.090863</td>
<td>0.001242</td>
<td>2.35136</td>
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</tbody>
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**Variance equation**

- R-squared: 0.090863
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- S.E. of regression: 0.001242
- Sum squared resid: 0.001242
- Log likelihood: 577.5295
- Hansen-Quinn criter.: -3.89924
- Durbin-Watson stat: 2.07626

Inverted AR Roots: 0.9
To sum up, all the GARCH (1,1), except the Iceland’s AR(1) coefficient, are statistically significant at a 5% significance level, since p critical is lower than the p-values of their coefficients and hence the null hypothesis is rejected. An important result is that the null is rejected in both Groups indicating that the participation in a rescue program didn’t affect the results.

### 8.5.3 EGARCH

The EGARCH model that was introduced by Nelson (1991) is another form of the GARCH model that tries to spot volatility clustering. More specifically EGARCH models can be found useful in cases where positive and negative shocks of equal magnitude do not affect volatility in the same way. The analysis of EGARCH (1,1) for the countries under scrutiny is presented below.

#### Table 7: EGARCH(1,1) results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.003072</td>
<td>0.000324</td>
<td>9.052182</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.01250</td>
<td>0.002988</td>
<td>-0.859371</td>
<td>0.3907</td>
</tr>
</tbody>
</table>

#### Variance Equation

| C(3) | 0.256414 | 0.030748 | -8.193708 | 0.0000 |
| C(4) | 0.116002 | 0.020078 | 5.129471 | 0.0000 |
| C(5) | 0.104063 | 0.019070 | 0.068771 | 0.0000 |
| C(6) | 0.091555 | 0.003008 | 22.0302 | 0.0000 |

#### EGARCH(1,1) results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.004007</td>
<td>0.000324</td>
<td>12.492182</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.01250</td>
<td>0.002988</td>
<td>-0.859371</td>
<td>0.3907</td>
</tr>
</tbody>
</table>

#### Variance Equation

| C(3) | 0.19958 | 0.030105 | -6.950751 | 0.0000 |
| C(4) | 0.091236 | 0.017462 | 5.272202 | 0.0000 |
| C(5) | 0.01250 | 0.012333 | -10.07575 | 0.0000 |
| C(6) | 0.092305 | 0.002673 | 337.5109 | 0.0000 |

#### EGARCH(1,1) results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.002224</td>
<td>0.000324</td>
<td>6.859371</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.01250</td>
<td>0.002988</td>
<td>-0.859371</td>
<td>0.3907</td>
</tr>
</tbody>
</table>

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| C(5) | 0.01250 | 0.012333 | -10.07575 | 0.0000 |
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#### EGARCH(1,1) results

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#### Variance Equation

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| C(5) | 0.01250 | 0.012333 | -10.07575 | 0.0000 |
| C(6) | 0.092305 | 0.002673 | 337.5109 | 0.0000 |
The conclusion, as far as both Groups is concerned, is that the coefficients of the EGARCH(1,1) model are statistically significant at significance level 5%, since the p-values are smaller than the p critical which is 0.05.

### 8.5.4 APARCH

One of the main positive aspects of the APARCH model is that it can capture asymmetry in return volatility. The results of APARCH (1,1) for all the countries under scrutiny are presented below.

#### Table 8: APARCH(1,1) results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000426</td>
<td>0.000216</td>
<td>1.971876</td>
<td>0.0486</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.039493</td>
<td>0.023043</td>
<td>1.713095</td>
<td>0.0865</td>
</tr>
</tbody>
</table>

#### Variance Equation

| C(0)     | -0.493260   | 0.846335   | -10.64545   | 0.0000|
| C(4)     | 0.137311    | 0.166277   | 8.135856    | 0.0000|
| C(5)     | -0.110663   | 0.010438   | 10.89000    | 0.0000|
| C(6)     | 0.058565    | 0.044373   | 10.50911    | 0.0000|

R-squared: 0.004254 Mean dependent var: 0.003188
Adjusted R-squared: 0.003762 S.D. dependent var: 0.010934
S.E. of regression: 0.019193 Akaike info criterion: -8.848259
Sum squared resid: 0.241568 Schwacch criterion: -7.381914
Log likelihood: 6437.953 Hannan-Quinn criter.: -6.107649
Durbin-Watson stat: 1.917486

Inverted AR Roots: 0.04

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000426</td>
<td>0.000216</td>
<td>1.971876</td>
<td>0.0486</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.039493</td>
<td>0.023043</td>
<td>1.713095</td>
<td>0.0865</td>
</tr>
</tbody>
</table>

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Sum squared resid: 0.241568 Schwacch criterion: -7.381914
Log likelihood: 6437.953 Hannan-Quinn criter.: -6.107649
Durbin-Watson stat: 1.917486

Inverted AR Roots: 0.04
For the countries regardless of whether they were under a rescue program or not, almost all APARCH(1,1) coefficients were found to be statistically significant at a 5% significance level. Those that were not significant are the C(3) coefficients of France, Portugal, and UK, while in the case of Germany, three out of seven C(3), C(4), and C(5) were not statistically significant since their coefficient's p values were bigger than the critical one.
8.5.5 TARCH

TARCH models are useful because they provide a threshold in both the conditional variance and the conditional mean of a time series, additionally they provide explanations regarding asymmetries as Hawg and Woo (2001) suggest. The below tables depicts the results of the TARCH (1,1) for the countries.

Table 9: TARCH(1,1) results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000217</td>
<td>0.000234</td>
<td>0.926062</td>
<td>0.3543</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.029390</td>
<td>0.042368</td>
<td>-2.04662</td>
<td>0.0410</td>
</tr>
</tbody>
</table>

Variance Equation

| C        | 2.238-06    | 0.041672   | 2.513-07    | 0.012060 |
| RESID(1)^2 | -0.006197  | 0.000133   | 0.054635    | 0.9585  |
| RESID(1)^2*RESID(1)^2 | 0.014472   | 0.001874   | 2.976647    | 0.0035 |
| GARCH(1)  | 0.819866   | 0.012186   | 77.85761    | 0.0000 |

R-squared: 0.001231 Mean dependent var: 0.000000
Adjusted R-squared: 0.000738 S.D. dependent var: 0.015500
S.E. of regression: 0.085149 Akaike info criterion: -5.915199
Sum squared resid: 0.095080 Schwarz criterion: -5.901574
Log likelihood: 0.001257 Hannan-Quinn crier: -5.929909
Durbin-Watson stat: 2.015260

Inverted AR Roots: -0.3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>3.123-07</td>
<td>0.923-08</td>
<td>3.446966</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESID(1)^2</td>
<td>0.008992</td>
<td>0.000343</td>
<td>2.42727</td>
<td>0.0127</td>
</tr>
<tr>
<td>RESID(1)^2*RESID(1)^2</td>
<td>0.0159179</td>
<td>0.018094</td>
<td>8.520141</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.890522</td>
<td>0.001140</td>
<td>82.15200</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 0.004362 Mean dependent var: 0.001998
Adjusted R-squared: 0.004370 S.D. dependent var: 0.001998
S.E. of regression: 0.027149 Akaike info criterion: -6.341279
Sum squared resid: 0.141919 Schwarz criterion: -6.324654
Log likelihood: 0.027149 Hannan-Quinn crier: -6.335178
Durbin-Watson stat: 2.035374

Inverted AR Roots: 0.0
As it can be deducted, the coefficients of the TARCH (1,1) are found to be statistically significant at a 5% significance level since the null hypothesis is rejected. The p-values of the coefficients are bigger than the critical value with an exception. This is the $\alpha_1$ coefficient for Germany, Ireland and UK which according to the results are not significant.

9. Comparison of the models and model selection

In order to conclude which model could forecast more efficiently the VaR for the scrutinized countries during the years followed the crisis, the Akaike’s (1974) info and the Schwarz’s (1978) info criteria are used. Both are based in the likelihood function. The likelihood can be increased if more parameters are included. One of the main concerns when adding parameters is to avoid overfitting. Overfitting appears when fitting a bigger model than the one needed to present the dynamics of the data. In order to avoid this phenomenon, both AIC and SBIC penalize the incorporation of additional terms with SBIC to have a stricter penalty term Brooks (2008). In addition when it comes to larger samples, AIC provides better results.
9.1 Akaike’s Information Criterion (AIC)

This criterion is one of the first used in order to evaluate a model’s quality for a given set of data and that is why is among the famous criteria of model selection. AIC can find which of the models is the optimal, since it accounts how intricate a model is. Bozdogan (1987). AIC puts more weight in contrasting the goodness of fit among models and the main idea behind it is that the lower the criterion, the better the model is. Algebraically, AIC is expressed:

\[
\text{AIC} = \ln(\sigma^2) + \frac{2k}{T} \tag{16}
\]

Where
\[
\sigma^2: \text{ is the residual variance}
\]
\[
k: \text{ is the sum of } p,q \text{ and } 1 \text{ and is the total number of parameters estimated}
\]
\[
T: \text{ is the sample size}
\]

The below Tables summarizes the results for the two Groups of countries that were analyzed in this paper.

<table>
<thead>
<tr>
<th>Akaike's Info Criterion</th>
<th>France</th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH (1)</td>
<td>-5,761</td>
<td>-5,904</td>
<td>-6,209</td>
</tr>
<tr>
<td>GARCH (1,1)</td>
<td>-6,057</td>
<td>-5,904</td>
<td>-6,503</td>
</tr>
<tr>
<td>EGARCH (1,1)</td>
<td>-6,099</td>
<td>-5,933</td>
<td>-6,541</td>
</tr>
<tr>
<td>APARCH (1,1)</td>
<td>-6,102</td>
<td>-5,939</td>
<td>-6,543</td>
</tr>
<tr>
<td>TGARCH (1,1)</td>
<td>-6,076</td>
<td>-5,935</td>
<td>-6,536</td>
</tr>
</tbody>
</table>

Table 10: Akaike's Info Criterion for the first Group

Regarding the first Group, the richest countries of Europe, the AIC criterion suggest that the optimal model for forecasting VaR during the latest financial crisis would be EGARCH (1,1) for France and APARCH (1,1) for Germany and UK.
Table 11: Akaike’s Info Criterion for the second Group

<table>
<thead>
<tr>
<th>Model</th>
<th>Portugal</th>
<th>Ireland</th>
<th>Iceland</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH (1)</td>
<td>-6,611</td>
<td>-6,238</td>
<td>-5,450</td>
</tr>
<tr>
<td>GARCH (1,1)</td>
<td>-6,801</td>
<td>-6,374</td>
<td>-6,304</td>
</tr>
<tr>
<td>EGARCH (1,1)</td>
<td>-6,816</td>
<td>-6,408</td>
<td>-4,825</td>
</tr>
<tr>
<td>APARCH (1,1)</td>
<td>-6,817</td>
<td>-6,414</td>
<td>-6,199</td>
</tr>
<tr>
<td>TGARCH (1,1)</td>
<td>-6,815</td>
<td>-6,395</td>
<td>-6,341</td>
</tr>
</tbody>
</table>

Table 12: Schwarz’s Criterion for the first Group

<table>
<thead>
<tr>
<th>Model</th>
<th>France</th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH (1)</td>
<td>-5,750</td>
<td>-5,890</td>
<td>-6,198</td>
</tr>
<tr>
<td>GARCH (1,1)</td>
<td>-6,043</td>
<td>-5,890</td>
<td>-6,489</td>
</tr>
<tr>
<td>EGARCH (1,1)</td>
<td>-6,082</td>
<td>-5,917</td>
<td>-6,525</td>
</tr>
<tr>
<td>APARCH (1,1)</td>
<td>-6,082</td>
<td>-5,919</td>
<td>-6,524</td>
</tr>
<tr>
<td>TGARCH (1,1)</td>
<td>-6,076</td>
<td>-5,918</td>
<td>-6,520</td>
</tr>
</tbody>
</table>

As far as the countries that were the most vulnerable and participated in a rescue program the AIC criterion reveals that the optimal model would be APARCH(1,1) when it comes to Portugal and Ireland and TGARCH (1,1) for Iceland.

Overall, although the optimal model was not the same in all countries or in the countries of the same Group, still APARCH (1,1) appears to be the most efficient. Therefore it could have a better predictive ability if it was used in VaR estimation.

9.2 Schwarz’s Bayesian Information Criterion (SBIC)

The Schwarz Criterion (1978) is another method of finding which the optimal model for a set of data is and it is based on the likelihood function. It is widely known and used although it is preferable for smaller sample sizes. Algebraically, SBIC is expressed

$$SBIC = \ln (\sigma^2) + \frac{k}{2} \ln T$$  \hspace{1cm} (17)

Where

\(\sigma^2\): is the residual variance

k: is the sum of p,q and 1 and is the total number of parameters estimated

T: is the sample size

The below Table 12 summarizes the results for the first Group of countries that was analyzed in this dissertation.
According to SBIC criterion, the decision regarding which model could provide more accurate VaR forecasts during the years of the financial crisis in the richest countries in Europe is not that clear. In the case of France both APARCH (1,1) and EGARCH(1,1) have the same SBIC coefficient. The same applies to the remaining two countries of the Group. For Germany both ARCH(1) and GARCH(1,1) seem to be accurate enough in order to be used in the VaR estimation. Last but not least, regarding UK once again APARCH (1,1) and EGARCH(1,1) could be used for the estimation of VaR.

<table>
<thead>
<tr>
<th>Schwarz's Criterion</th>
<th>Portugal</th>
<th>Ireland</th>
<th>Iceland</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH (1)</td>
<td>-6,600</td>
<td>-6,227</td>
<td>-5,440</td>
</tr>
<tr>
<td>GARCH (1,1)</td>
<td>-6,788</td>
<td>-6,360</td>
<td>-6,290</td>
</tr>
<tr>
<td>EGARCH (1,1)</td>
<td>-6,799</td>
<td>-6,391</td>
<td>-4,800</td>
</tr>
<tr>
<td>APARCH (1,1)</td>
<td>-6,797</td>
<td>-6,304</td>
<td>-6,180</td>
</tr>
<tr>
<td>TGARCH (1,1)</td>
<td>-6,799</td>
<td>-6,379</td>
<td>-6,320</td>
</tr>
</tbody>
</table>

Table 13: Schwarz's Criterion for the second Group

As far as the second Group is concerned, in the countries most exposed to the financial crisis the optimal models appear to be EGARCH (1,1) for Ireland and TGARCH(1,1) for Iceland. In the case of Portugal, EGARCH (1,1) and TGARCH(1,1) seem to have the same SBIC coefficient and therefore the dissertation suggests that both models could be used in VaR forecasting.
Conclusions

Value at Risk is a very important key in order to measure market risk. One of the main objectives of this dissertation was to find out which model could have offered accurate predictions of the VaR during the latest financial economic crisis. In addition it was investigated whether the size of the economy is an important factor of VaR forecasting. The dissertation reveals that Historical Simulation couldn’t be a sufficient method of forecasting VaR since according to backtesting procedure, it was rejected in almost all countries. Based on the Information Criteria, there is no apparent model that is optimal for all the scrutinized countries and could be used in the VaR forecasting. What's more evidence prove that the condition of the economy doesn't affect the model selection. Regarding SBIC, in many cases the comparison showed that for the same country more than one model could be applicable. The use of AIC on the other hand, provides more clear results regarding which model could be used in order to forecast more accurately the VaR for the European Stock Exchanges during the latest financial crisis with EGARCH (1,1) and APARCH(1,1) to be the prevailed ones in both Groups. The efficient measurement of market risk is a vital priority for both corporations and nations; that is why it is essential to estimate the optimal model in order to moderate the consequences of the market movements.
References


Schwert, ., (1989. Why does stock market volatility change over time?. *J Fin 44, 1115–1153*
Statistisches Bundesamt, Available at: https://www.destatis.de/EN/Homepage.html [Accessed 22 August 2016]


