Exchange Rate Fluctuations Forecasting

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Abstract

Exchange rates fluctuate due to the continuous changes in supply and demand for different currencies. Also important factors that determine the value of a currency are the GDP growth, the inflation, and the output growth of a country.

Consequently forecasting exchange rates is important not only for a profitable FX trading strategy but also for macroeconomic reasons. Exchange rates contain information about the inflation, the level of income (population purchasing power), and the output growth of a specific set of countries. There are two broad methods for determining future exchange rates, called “fundamental analysis” and “nonstructural analysis”. Fundamental analysis is based in solid economic theories about growth, interest rates etc, hence it is a theoretical way of trying to predict future exchange rate movements. Nonstructural analysis is not based in economic theories, but instead uses the information hidden in the past values in order to make predictions for the future. Hence nonstructural analysis simply uses advanced statistics in order to predict the movements of exchange rates. Some of these models are the vector autoregression models, the various Markov models, the non-parametric models etc. However, an overview of the research suggests that while some successful predictions are possible, no one type of analysis can yield accurate forecasts for different currencies for long horizons.

In this work structural (Purchasing Power Parity, Interest Rate Parity) and nonstructural models were applied to study three very important exchange rates, euro, pound sterling, and yen versus the US dollar. Our work shows that Stochastic processes seem to outperform the other analysis models (Vector Auto-Regression process) during relatively long intervals (1 year). For time horizons (of the order of a month) VAR modelling can give more accurate results. A combination of both fundamental analysis and nonstructural analysis is more effective for analyzing exchange rates time-series data.
Fundamental and Non-Structural analysis models

A variety of fundamental-analysis models has been applied in exchange rate forecasting. Behavioral equilibrium models, interest rate parity models, portfolio balance models etc were implemented in this process. However, despite a history of research beginning in the late 1970s, it appears that the monetary model is not more effective than a relatively simple stochastic model which is based in the Black Scholes equation, when the volatility of the exchange rate is successfully calculated, and this important parameter does not fluctuate rapidly.

Despite the many efforts, accurate exchange rate forecasting becomes more and more difficult as the time horizon increases. Despite the use of modern non-structural analysis after the failure of fundamental-analysis models, there is not still sufficient evidence that any exchange-rate model can consistently out-perform a “random walk” in the long run. Various reasons have been offered to explain the difficulty in performing accurate exchange rate forecasting, such as bad data quality, time-variance between the relationship of exchange rates to their fundamentals, even behavioral factors. It is a historical event that fear, greediness, and very optimistic expectations based on distorted information by the investors move exchange rates many times in the opposite direction. Wrong political decisions and “corrections” by the governments in the interest rates, affected dramatically the exchange rates. Governments and the central banks actions have caused exchange rate panic many times in the past. A historical example is the peso in Argentina, which was fixed against the U.S. dollar. Argentina’s government could not follow the growth of the United States and irregular monetary supply created huge liquidity problems. After Brazil devalued its real in 1999, foreign investors decided to invest in neighboring Brazil than in Argentina. Argentina’s foreign investments and exports almost stopped. In addition to that, panic moved large amounts of money out of the country’s borders after being converted to US dollars. Negative growth, liquidity problems and large cash withdrawals from the large banks of Argentina, caused a very big social and financial crisis. A second wave of the crisis caused the debt of the country to increase sharply, and inflation reached astronomical units. Argentina could not pay the interests on its large debt and finally requested the help of the IMF in order to stabilize its economy. We have to note that the recovery took more than 10 years.
Chapter 1

Purchasing Power Parity

Financial theory suggests that there is a kind of long-run relationship between the ratio of relative prices and the exchange rate between two countries. The purchasing power parity theory (PPP) states that a given representative basket of goods and services, should cost the same wherever it is bought, when converted into a common currency. In other words PPP implies that the ratio of relative prices in two countries and the exchange rate between them should be close. If this is not the case, assuming zero transaction costs, it would be profitable to buy goods in one country, sell them in another, and convert the earnings back to the currency of the original country.

According to the law of one price, identical goods should (under certain conditions) have the same price in two different countries at the same time. The absolute PPP exchange rate equals the national price levels in two countries, if expressed in a common currency at that rate, so that the purchasing power of one unit of the currency would be the same in the two countries. The term Purchasing Power Parity was first used by Cassel (1918), but the roots of this theory go back in the fifteenth century.\(^1\)

In its absolute version, the purchasing power of different currencies is equalized for a given basket of goods. In the relative version, the difference in the inflation rates, is equal to the percentage change of the exchange rate. The best-known and most-used purchasing power parity exchange rate is the ”international dollar”. In other words, PPP is the amount of a certain basket of basic goods which can be bought in the given country with the standard international currency.

PPP exchange rate (real exchange rate) fluctuations are mostly due to different rates of inflation between the two economies. Aside from this volatility, consistent deviations of the market and PPP exchange rates are observed. For example prices of non-traded goods and services are usually lower where incomes are lower.

An important fact is that the PPP exchange-rate calculation is controversial because of the difficulties of finding comparable baskets of goods to compare purchasing power across countries. Estimation of purchasing power parity is complicated by the fact that countries do not simply differ in a uniform price level, rather the people in different countries consume different baskets of goods in general. Demand is higher for certain types of goods and this finally increases their prices.

Apart from these it is necessary to make adjustments for differences in the quality of goods and services. Additional statistical difficulties arise with multilateral comparisons when (as is usually the case) more than two countries are to be compared. Also when PPP comparisons are to be made over some interval of time, proper account needs to be made of inflationary effects.

There are, actually, different versions of the Law of One Price. There is a strong absolute version and a weaker relative version. Both can be applied to individual products and to price indices. Let’s start with the strongest version of all. Suppose we have a large number N of individual products consumed and produced in America and Britain. We let the sub-index i denote the type of product, (so i ranges from 1 to N), and the sub-index t denote time. Then the absolute version

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\(^1\)The Swedish economist Gustav Cassel was a founding member of the Swedish School of economics. His most important work is on the theory of interest rates (The Nature and Necessity of Interest, 1903)
of the Law of One Price for each individual good \( i \) implies:

\[
P_{Bi,t} = S_t \cdot P_{Ai,t}
\]

\( P_{Ai,t} \): price of good \( i \) in America at time \( t \) (in dollars)

\( P_{Bi,t} \): price of the same good in Britain at the same time

\( S_t \): the nominal exchange rate of the US dollar (The price in GBP for purchasing one dollar).

This equation imposes a restriction on the price levels of the same good in different countries. Instead, the relative version of the Law of One Price imposes a restriction on the changes in these price levels, more specifically:

\[
\frac{S_{t+1} \cdot P_{Ai,t+1}}{P_{Bi,t+1}} = \frac{S_t \cdot P_{Ai,t}}{P_{Bi,t}}
\]

The relative version tells us that the deviation between the prices of some good in the two countries is relatively constant through time.

In order to find the law of PPP from the law of one price we have to move from the microeconomic level to the macroeconomic characteristics of a country. There are various indices published (Consumer Price Index CPI, the producer price index, the GDP deflator). These indices show different aspects of an economy but can be used cautiously in order to get an image of the PPP for a country. If we use the CPI (constructed as a weighted average of the prices of individual groups of products), then for USA:

\[
P_{A,t} = \sum_{i=1}^{N} \alpha_i \cdot P_{Ai,t}
\]

\[
\sum_{i=1}^{N} \alpha_i = 1, \alpha \geq 0
\]

Similarly for Britain, if the law of one price is valid in its absolute form, we have:

\[
P_{B,t} = \sum_{i=1}^{N} \alpha_i \cdot P_{Bi,t} = \sum_{i=1}^{N} \alpha_i \cdot (S_t P_{Ai,t}) = S_t \cdot \sum_{i=1}^{N} \alpha_i \cdot P_{Ai,t} = S_t \cdot P_{A,t}
\]

Now if we take the logarithmic form of this equation, (the lower case letters express logarithms) we can find the absolute version of PPP in logarithmic form.

\[
s_t = p_{B,t} - p_{A,t} \quad (E1)
\]

**Definition :** \( x = \ln(X), \) \( s_t = \ln(S_t), \) \( p_{A,t} = P_{A,t} \)

The relative version is also straightforward:

\[
s_{t+1} - s_t = (p_{B,t+1} - p_{B,t}) - (p_{A,t+1} - p_{A,t}) \quad (E2)
\]

The absolute version of the law of one price is the strongest condition. All other versions are implied from this relationship. ²

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Real effective FX rate

We defined before the nominal exchange rate between two countries, (equations E1, E2), consistent with the absolute or relative PPP. The real exchange rate is the difference between the nominal exchange rate and the price indices of the two countries:

\[ q_t = s_t - (p_{B,t} - p_{A,t}) \]

It expresses the deviation from PPP between the two countries we examine (Britain and US as we said before in our example). We can clearly see that this is a measure of the competitiveness of a specific country. When the real exchange rate increases, this means that dollar strengthens and the higher price of the $ US is only partially offset by differences in price developments \( p_{B,t} - p_{A,t} \), hence America has become less competitive. 3

Usually countries are interested in their competitive position in general. The Real Effective Exchange Rate expresses a weighted average of the bilateral real exchange rates. It is an indicator of the competitiveness of domestic relative to foreign goods and the demand for domestic and foreign currency assets.

The following scheme shows the relation between exchange rates and price indices for 55 different countries between 1960 and 2001. There is clearly a linear relationship between the CPI and the FX Rate (US dollar is the benchmark currency). The scheme is taken from the book International Economics: Theory, Application, and Policy, 2007, by Prof Charles van Marrewijk.

Fig1: US dollar exchange rates and Consumer Price Indexes for 55 countries between 1960 and 2001

This relationship is expected. The logarithmic form, of the law of relative PPP imposes for this case:

\[ s_{2001} - s_{1960} = (p_{c,j,2001} - p_{c,j,1960}) - (p_{US,2001} - p_{US,1960}), \quad c_j = 1, \ldots, 55 \]

Chapter 2

Interest rate parity

Interest rate parity relates interest rates and exchange rates. Interest rate parity is a non-arbitrage assumption. It simply states that an investor cannot profit from borrowing in the risk free rate in one country, invest the money in another at the domestic risk free rate (after converting the money in the other country’s currency) and simultaneously lock in a risk-less profit by entering a long position in a futures contract, in order to convert the money back to the first currency at the end of his investment horizon. This argument leads to the cost of carry model, a slightly more general model, used to find the forward price of any commodity:

\[ F = S \cdot e^{(r + S - c)t} \]

where \( F \) is the forward price, \( S \) is the spot price, \( r \) is the risk free interest rate, \( c \) is the convenience yield, and \( t \) is the time to delivery of the forward contract (years).

For currencies there is no storage cost, and \( c \) is interpreted as the foreign interest rate. Hence the difference between the (risk free) interest rates paid on two currencies should be equal to the differences between the spot and forward rates.

If we assume zero transaction costs, then arbitrage is possible if the equation doesn’t hold. If \( F \) is very high, then it is profitable to sell the forward currency, if the inverse holds then it is profitable to buy the forward currency.

The Covered Interest Parity equation demands that \( f_t - s_t = r_t - r^*_t \), \( f_t, s_t \) are the log of the forward and spot prices of the domestic in terms of the foreign currency at time \( t \), and \( r, r^* \) are the domestic and foreign risk free rates. For example if the dollar-yen interest rate gap is 3%, the interest rate for dollar is 5% and the interest rate for yen is 2%, the forward contract for dollar-yen will imply a 3% premium over today’s exchange rate for yen. The interest rates offered by two countries and the exchange rate of their currencies cannot deviate from the non-arbitrage condition mentioned before.

Uncovered Interest Parity introduces the “Forward Rate Unbiasedness”. Forward Rate Unbiasedness states that the forward rate of foreign exchange should be an unbiased predictor of the future value of the spot rate. This can be expressed mathematically \( s^e_{t+1} - s_t = r_t - r^*_t \). Here \( s^e_{t+1} \) is the expectation, made at time \( t \) of the spot exchange rate that will prevail at time \( t+1 \) in the future. In the data sample we examined, Yen gained value over dollar during 2008. The Japanese risk free rate was 0.5% and the American risk free interest rate was above 2.18% during 2008. Yen actually appreciated even more than the UIP model predicts. At the beginning of 2008 1$ = 109.58 ¥ and at the end (12/31/2008) 1$ = 90.79 ¥.

This cornerstone condition for efficiency in the FX Market states that the difference between the rates of return on two currencies, is the interest rate gap plus the expected appreciation. In our previous example the 12-month dollar interest rate for 2008 was 2.18% while the 12-month yen interest rate is 0.5%, then according to the theory, markets must expect the yen to appreciate 2.18-0.5=1.68%.
The famous “Fama regression” (Fama-1984) showed that UIP is often violated in the data. Research shows that the actual appreciation does not follow the forward rate. It is the currency with the high interest rate that tends to appreciate, not the one with the low interest rate. Maybe because many investors keep their capital in the currency with the higher interest rate and show unwillingness to invest in the foreign currency with the lower interest rate, ignoring the fact that a higher interest rate is usually associated with higher inflation. In a related paper, using forward rates up to one year Clarida and Taylor (1997) have already shown that the random walk model of exchange rate forecasting can be outperformed. Long maturity Forward contracts show stationarity and seem to follow the Covered Interest Parity Rule.1

![Image Description: Fig CIP-1: Difference in interest rates between UK and US plotted with FX Rate $./£. There are periods (1972-1978, 1987-1990) when increasing difference in interest rates between the two countries, caused a fall of the pound after a period of 1-2 years.]

1 Ref [13][5][6][16]
Chapter 3

Deviations from Purchasing Power Parity Theory

Purchasing power parity as a model is considered today obsolete. There have been many tests of this law the last 5 decades with various results. Early empirical tests (until 1974) approved the theory and was shown that the model holds for various currencies in the long run. Deviations were found for relatively short periods. (see Sarno and Taylor Exchange Rate Economics Ch-3 (Cambridge University Press, 2003)).

The equation one has to test is:
\[ s_t = \gamma_1 + \gamma_2 \cdot p_{at} + \gamma_3 \cdot p_{bt} + u_t \]

If \( \gamma_1 = 1, \gamma_3 = 1 \), absolute PPP holds. Using the difference \( s_t - s_{t-1} \) one can test for the relative PPP.

Early studies did not use dynamics to distinguish between long-run and short-run effects, hence their results did not prove the PPP hypothesis. Frenkel -1978 used data from high inflation countries and with a more detailed test found results near the PPP values, suggesting that PPP holds for relatively long periods. Another technical issue in the testing of this equation is that usually cointegration and stationarity problems arise (Engle and Granger (1987)). When relatively short time intervals are used, the test has low power, and when relatively large intervals are examined, there are large changes in the FX rate dynamics. Also one has to take into account the different regimes that historically occurred for the FX rate between different countries (Gold Standard, Bretton Woods System, floating exchange rates which occurs today).

Another basic theoretic problem is that we cannot safely assume that simply prices determine exchange rates, but both prices and exchange rates are determined simultaneously in a large economic system. Prices and FX rates are cointegrated in a significant degree.

George Alessandria -2007 showed that deviations from the law of one price in tradable goods are an important source of violations of absolute PPP across countries. **Major U.S. exporters for example ship the same good to low-income countries at lower prices. Alessandria showed clearly that a doubling of income per capita is associated with a 43 percent higher price level.** The Law of one price holds only for tradable goods across countries and PPP is violated for non-tradable goods most of the times.

1Ref [13][5][7][8]
2Ref [15][1]
3Ref [1][2]
The model of Harrod (1933), Balassa (1964), and Samuelson (1964), the HBS model, states that tradable goods drive the changes in international relative wages.  

Fig3: Natural Logarithm of (DM/$) is plotted against the U.S. Consumer Price Index. Data refer to period after Ref [15]
the reunification of Germany (from 1992 to 2008). After Germany entered Euro-Area the value of DM was fixed to $1=1,995DM. In our analysis we use this fixed rate after 1999. Source: IMF database

Fig4: Natural Logarithm of (DM/$) is plotted against the German Consumer Price Index. Data refer to period after the reunification of Germany (from 1992 to 2008). Source: IMF database

Fig5: Inflation in Germany (percentage term). Data refer to the period after the reunification of Germany (from 1992 to 2008). Source: IMF database
Fig 6: EVIEWS Regression. The absolute PPP theory does not hold, but the sample is relatively small. The German FX-Rate is relatively stable and tied to the American dollar. Data refer to the period after the reunification of Germany (from 1992 to 2008).

Fig 7: Absolute PPP cannot be approved from our results.
Fig 8: Inflation in 3 countries (Japan, USA, UK). Notice the negative values during the last 2 decades in the Japanese inflation.

Fig 9: The Yen has appreciated significantly during this period. This reflects the strength of GDP growth and the high exporting activity of Japan. Data refer to the period from 1961 to 2008.
Fig 10: The Yen has appreciated significantly during this period compared to US dollar. During the 60s the exchange rate was 3$ per 1000 ¥, today the exchange rate is almost 9$ per 1000 ¥. Notice that periods of high inflation (1970s) are connected with the depreciation of the Japanese currency.

Fig 11: Absolute PPP does not hold for Japan.
Tests for Relative PPP

Now we are going to test some countries in order to find if the relative purchasing power theory holds. By comparing Observed FX Rates and Theoretical rates (according to PPP), we can find if the currency is overvalued or undervalued. Hence if PPP holds we can predict its future movement towards its intrinsic (fair) value. Data were downloaded from the IMF, the FED Reserve, the Bank of England, the Bank of Japan, and the European Central Bank. In order to find about the validity of the theory we will test UK, France, Greece, Japan, Germany and the Euro Area.

Many economists are supporters of the idea of tracking prices of Big Mac Burger or Starbucks coffee across different countries to find the intrinsic fair value of a currency relative to U.S.$. The idea is that a cup of coffee or a big Mac should cost the same everywhere (Law of one price). Bloomberg also uses this index to compare different currencies and find if these are over/under valued. There are big supporters of the MacBurger idea around the globe, but this index can lead to completely wrong conclusions, because in many places in the world people prefer the traditional way of nutrition. Another issue is that different tax policy leads to completely different prices among various countries. In some countries Burgers and Starbucks coffee are considered luxury goods, consequently they are heavily taxed. BigMac data about PPP of different countries are included in the appendix.

The relative version of PPP was described before and is calculated by the relation.

\[ s_{t+1} - s_t = (p_{F,t+1} - p_{F,t}) - (p_{A,t+1} - p_{A,t}) \]

The exchange rate is the €/$, where the $ is considered the asset currency we are buying with €. In the following graphs the x-axis represents the difference \( \ln(\frac{P_{A,t+1}}{P_{A,t}}) \) for the US economy (Foreign Currency) and the y-axis the difference \( \ln(\frac{P_{F,t+1}}{P_{F,t}}) \) for France (Domestic Currency). In order the relative PPP to be valid the graph should have a slope of 45 degrees. We write again the equation (E2) in a more clear form: \( \ln(\frac{S_t}{S_{t-1}}) = \ln(\frac{P_{F,t+1}}{P_{F,t}}) - \ln(\frac{P_{A,t+1}}{P_{A,t}}) \) and \( \ln(\frac{P_{F,t+1}}{P_{F,t}}) = \ln(\frac{S_t}{S_{t-1}}) + \ln(\frac{P_{A,t+1}}{P_{A,t}}) \).

Now the linear form \( Y = a + b \cdot X \), \( b = 1 \) is evident.
Relative PPP for France.

Fig12: It is evident that Relative PPP holds for France. Data were downloaded from the International Monetary Fund (IMF) Database.

Relative PPP graph for Germany. Germany was divided after World War 2, hence there are not available data for Eastern Germany in the IMF Databases. We found some data in other sources but we didn’t find them to be reliable.

Fig13: It is evident that Relative PPP does not hold for Germany. The sample is relatively small 1991-2008. IMF data do not include Eastern Germany statistics.
Relative PPP graph for Greece. Greece had a strange inflation curve as we will show later. Inflation was adjusted after Greece entered the Euro-zone.

Fig14: It is evident that Relative PPP does not hold for Greece.

Fig14b: Notice the high inflation rates in Greece because of the excessive monetary supply.
Relative PPP graph for the United Kingdom. UK inflation curve is very similar to the U.S. curve. The relative PPP holds for the U.K. and the pound sterling.

Fig14c: It is evident that Relative PPP holds for Britain.
Fig UK-1: Exchange rate £/$, actual and calculated according to the PPP. Dollar is the asset currency we buy with pounds.

In the previous graph, we see that when the Blue line (Observed FX rate) is placed over the Red line (PPP theoretical), the observed values are lower than the theoretical values (according to PPP). The currency is undervalued. Hence one expects in the future the pound to strengthen. This result is in accordance with the results of the various exchange rate analysts. (See the appendix)
Relative PPP graph for Japan and the YEN. The relative PPP holds for Japan with a small deviation.

Fig14d: It is evident that Relative PPP holds for Japan.
Fig JAP-1: Exchange rate ¥/$, actual and calculated according to the PPP. Dollar is the asset currency we buy with Yen.

In the previous graph, we see that when the Blue line (Observed FX rate) is placed over the Red line (PPP theoretical), the observed values are lower than the theoretical values (according to PPP). The currency is undervalued. After 1995 Yen is overvalued. This result is in accordance with the results of the various exchange rate analysts, who expect the Japanese currency to fall relatively to the US dollar. (See the appendix)

Often when a government decreases the interest rates, the currency usually depreciates in contrast to what UIP theory predicts. UIP violations were found in the data, but in general interest rate parity holds. Another case is that governments with high external debt, facing economic slowdown, usually decrease the interest rates and increase the monetary supply in order to stimulate the economy, and collect taxes. This sharply depreciates their currency, and this is the case with the $/Y. Dollar is losing value the last 2 years on average, but many analysts believe that yen will finally go down near its PPP value the next period.
Relative PPP graph for Euro-zone. The relative PPP is calculated with a relatively small sample 1998-2008. Source is EuroStat and the data contain the HCPI weighted average index of the countries entered euro area till 2006. EuroStat publishes this index for U.S.A. too, in order to be comparable.

Fig15: It is evident that Relative PPP does not hold during this decade.
Fig EU-1: Exchange rate €/$, actual and calculated according to the PPP. Dollar is the asset currency we buy with euros. After 2007 € is overvalued.
Determinants of FX rates

Exchange rates fluctuations are caused by actual monetary flows but also by expectations. GDP changes, changes in growth, changes in inflation and in interest rates affect FX Rates. Also changes in public debt and trade deficits, changes in unemployment all cause different expectations in market makers in Foreign Exchange. News released almost every day, yearly or quarterly play a major role in this global trillion dollar market. A basic fact that cannot be ignored in this market is that large banks have an important advantage; they can see their customers’ order flow.

Today FX rates are balanced in a floating exchange rate regime. Previous regimes were more or less centrally controlled. The developed countries in the past agreed that the liberal international economic system required governmental intervention. This was the basic reason for a global stable monetary policy after World War II. The Bretton Woods system was the first example of a fully negotiated monetary order intended to govern monetary relations among independent nation-states. The planners at Bretton Woods established the International Monetary Fund (IMF) and the International Bank for Reconstruction and Development (IBRD), which today is part of the World Bank Group. These organizations became operational in 1945 after a sufficient number of countries had ratified the agreement.

The following theories explain the fluctuations in FX rates in a floating exchange rate regime (In a fixed exchange rate regime, FX rates are decided by the governments):

We described previously the theories of International parity conditions: Relative PPP, interest rate parity etc. The above theories provide logical explanation for the fluctuations in exchange rates, yet these theories many times are not precise as they are based on challengeable assumptions (e.g., free flow of goods, services and capital) which often do not hold in the real world.

Balance of payments model: This model focuses largely on tradable goods and services. It states that during periods of high public deficit, the currency of a country is depreciated. Its basic drawback is that it ignores the increasing role of global capital flows. It failed to provide a solid explanation for the appreciation of dollar during the 80s and 90s while the US current account deficit was soaring.

Another theoretical attempt to explain FX Rates fluctuations is the Asset market model. The model views currencies as an important asset class for constructing investment portfolios. The models assumptions are based in modern portfolio theory. Assets prices are influenced mostly by people’s willingness to hold the existing quantities of assets, which in turn depends on their expectations on the future worth of these assets. The asset market model of exchange rate determination states that “the exchange rate between two currencies represents the price that just balances the relative supplies and demand for assets, denominated in those currencies.”

None of the models developed so far succeed to explain FX rates levels and volatility in the longer time frames. For shorter time frames (few trading days) various algorithms have successfully predicted prices. It is understood from the above models that many macroeconomic factors affect the exchange rates and finally currency prices are a result of dual forces of demand and supply. Supply and demand for any given currency, and thus its value, are not influenced by any single element, but rather by several. These elements generally fall into three categories: economic factors, political conditions and market psychology.

\[\text{Ref [2], [14], Wikipedia}\]

\[\text{Ref [14], Wikipedia}\]
Chapter 4

Nonstructural Models implemented

In contrast to fundamental-analysis models, nonstructural analysis is not based in Macroeconomic Theories. These methods are based in pure mathematics, stochastic processes, game theory, and other advanced statistics models. Some types of these models include vector autoregression models, Markov switching models, Kolmogorov Theory in Statistical Mechanics, neutral networks theory, signal theory. Almost every model found to explain phenomena in the real world, was applied in the FX Market. The limited volume of this work, of course, does not allow us to explain every theory applied in the past. Instead we will apply the VAR modeling in order to test whether there are lead-lag relationships for the returns to three exchange rates against the US dollar. We will test three major currencies Euro, GBP, JYen. Our work will try to answer the fundamental question: “Given a set of data, with well determined characteristics, can we find a plausible model which explains our observations?”

In time series modeling, a researcher tries to connect future values of a financial variable with the values the variable took previous times. The basic assumption is that the values of the variables contain the past information and there is no need for an economic theory to explain those values. In other words the researcher does not care about explaining the data sample, rather he studies the sample and tries to find correlations between past and present values. Time series models are very useful in the FX trading, and many large market participants use them in order to make short term predictions. This is because the structural models are not usually effective for out of sample forecasting, and as we saw earlier there are many factors affecting the FX market, which are measured quarterly or yearly. An important class of time-series models proposed by Box and Jenkins (1976) are the Autoregressive Integrated Moving Average Models (ARIMA). 1

Fundamental Characteristics of time-series

An important concept when somebody examines time series data is the stationarity of the data. A series is strictly stationary if the distribution of its values remains stable with time. This means that the probability that the variable examined to be found in a particular interval is the same at any time in the past or the future.

A stationary process should have a constant mean, a constant variance, and a constant auto-covariance structure. This is defined mathematically as :

\[ E(y_t) = \mu \tag{a1} \]

\[ E(y_t - \mu)(y_t - \mu) = \sigma^2 \tag{a2} \]

1Ref [14][1][13]
\[ E(y_{t_1} - \mu)(y_{t_2} - \mu) = \gamma_{t_2 - t_1} \quad (a3) \]

Equation (a3) defines the autocovariance function, which is the same with the variance of \( y \) when \( t_2 = t_1 \).

Similarly the autocorrelation is defined as \( \tau_s = \frac{\gamma_s}{\sigma^2} \), \( s = 0, 1, 2, 3,... \). The series of autocorrelation function has the standard property its values to lie in the interval \([-1,+1]\). If \( \tau_s \) is plotted against \( s=0,1,2.. \), the correlogram is obtained.

**White noise definition**

A white noise is a process without discernible structure. It’s definition is:

\[ E(y_t) = \mu \quad (wn1) \]

\[ E(y_t - \mu)(y_t - \mu) = \sigma^2 \quad (wn2) \]

\[ \gamma_{t-r} = \begin{cases} \sigma^2 & \text{if } t = r, \\ 0 & \text{elsewhere} \end{cases} \]

The white noise process has constant mean and variance, but zero autocorrelation, except at lag zero. *Each observation in the sample is completely uncorrelated with the others.* If \( \mu = 0 \) we have a zero mean white noise process. If it is further assumed that \( y_t \) is distributed normally, then the sample autocorrelation coefficients are also approximately normally distributed, \( \hat{\tau}_s \approx N(0, \frac{1}{T}) \). Here \( T \) is the size of the sample (number of observations) and \( \hat{\tau}_s \) is the autocorrelation coefficient at lag \( s \) estimated from the sample. **Hence a 95\% non-rejection region can be introduced by the known interval \([-1.96 \cdot \sqrt{1/T}, 1.96 \cdot \sqrt{1/T}]\).** If the sample autocorrelation coefficient lies outside this interval then the Null Hypothesis, that the true value of the coefficient at lag \( s \), is zero can be rejected. \(^2\)

**Autoregressive and Moving Average Processes**

A moving average process of order \( q \) contains only white noise terms: \( y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \theta_3 u_{t-3} + .... + \theta_q u_{t-q} \). This process is known as MA(q). It can be written using the lag operator as:

\[ y_t = \mu + \sum_{i=1}^{q} \theta_i L^i u_t + u_t \]

or in a more compact notation \( y_t = \mu + \theta(L) u_t, \theta(L) = 1 + \theta_1 L + \theta_2 L^2 + ... + \theta_q L^q \). A MA process is described by a constant mean, constant variance and zero auto-covariances after lag \( q \).

An autoregressive process of order \( p \) contains only lags of the variable and a white noise term:

\[ y_t = \mu + u_t + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + .... + \phi_p y_{t-p} \]

\(^2\)Ref [1],[2],[5]
Stationarity Condition

The stationarity condition of an AR(p) process is found after solving its characteristic equations.

\[ \phi(L)y_t = \mu + u_t \]

In this case \( \phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3 - \ldots - \phi_p L^p \). The equation leads to a polynomial \( 1 - \phi_1 z - \phi_2 z^2 - \phi_3 z^3 - \ldots - \phi_p z^p - \mu = 0 \). In order the stationarity condition to hold, all the roots should lie outside the unit circle.

A random walk has the form \( y_t = \mu + q y_{t-1} + u_t \) with \( \mu < 1 \). Its characteristic polynomial is \( 1 - q z = \mu \) and finally if \( z = \frac{1-\mu}{q} < 1 \) the process is not stationary.

Any stationary series can be decomposed into the sum of two unrelated processes, a purely deterministic part and a purely stochastic part. Hence an ARMA process can be obtained if we combine an AR and a MA process.

Vector Autoregressive Models (VAR)

VAR models were popularized in econometrics by Sims during 1980. VAR are regression models with more than one dependent variables. VAR is a hybrid between the univariate autoregressive model AR, and the simultaneous equation models. VAR can be considered an alternative to large-scale simultaneous equations structural models. A typical VAR model can be expressed mathematically:

\[
\begin{align*}
y_{1t} &= \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + u_{1t} \\
y_{2t} &= \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + u_{2t}
\end{align*}
\]

or in matrix form:

\[
\begin{pmatrix}
y_{1t} \\
y_{2t}
\end{pmatrix} =
\begin{pmatrix}
\beta_{10} & \beta_{11} \\
\beta_{20} & \beta_{21}
\end{pmatrix}
\begin{pmatrix}
y_{1t-1} \\
y_{2t-1}
\end{pmatrix}
+ \begin{pmatrix}
\alpha_{11} \\
\alpha_{21}
\end{pmatrix}
\begin{pmatrix}
y_{2t-1} \\
y_{1t-1}
\end{pmatrix}
+ \begin{pmatrix}
u_{1t} \\
u_{2t}
\end{pmatrix}
\]

Advantages of Vector Autoregressive Models

VAR models have several advantages over univariate time series models or simultaneous equations structural models. VAR models combine both characteristics of univariate and structural modeling. Firstly all variables in the VAR models are considered endogenous. Secondly VAR models allow the value of a variable to depend on more than its own lags. In our model we combined the time-series in Euro, Pound Sterling, Yen because we saw that the time series are highly correlated between them. VAR models offer rich structure, hence it is possible to capture more features of the sample. In our case we captured information on positive correlation between Pound and Euro and lower positive correlation between Yen-Euro and Yen-Pound. We have to note here that a potential investor can take advantage of this correlation for hedging a long position in Euros or Pounds with a properly weighted long position in Yen, based on the correlation matrix.

Historically the forecasts generated by VAR models were often better than traditional structural models. Large scale structural models often perform badly in terms of their out of sample accuracy. A series of publications by Sims -1980 brings evidence about the weaknesses of structural models. McNees (1986) showed that VAR models managed to predict the U.S. unemployment rates more effectively compared to other structural methods used in the past. 3

3Ref [1][6][7]
VAR models are not based upon any underlying economic theory most of the times. This can be an advantage, when we see a correlation between different variables, but the reasons can be far too many to be included in a structural model based on a solid economic theory. The problem with structural modeling is that different explanatory variables are measured in different time periods or even their values are sometimes manipulated for political reasons. (Unemployment rate, budget deficit of a country, GDP growth).
Chapter 5

Presentation of time-series Data

The following 6 graphs show the properties of the sample. Data were downloaded from the FED Reserve Website. They are daily observations of FX Rates for Euro, GBP and Yen between 4 January 1999 and 31 December 2008. There is strong evidence from the graphs that Euro and Pound show high positive correlation. We also show the graph of continuously compounded returns, which were used in our whole analysis because of their time-consistency.

![Graph of Euro and GBP Values vs Time (Day of Trading)](image)

FigEU-GBP-1: Euro and GBP Values vs Time (Day of Trading)
FigEU-GBP-2: Logarithmic Returns Values vs Time (Last 200 Days of Trading)

FigYEN-1: Yen Values vs Time (Day of Trading)
Fig16: We see here the high positive correlation between Euro and Pound. An important fact is the lower correlation between Yen and the other currencies.
Histograms of Continuously Compounded Returns

Histogram of Series Statistics for Logarithmic Returns. The distribution shows kurtosis "fat tails" but we can assume normality of Returns. We know that a normal distribution is not skewed and its kurtosis=3. The Null Hypothesis for the Bera-Jarque statistic is that the series examined is normal. In our case we reject the Null Hypothesis of normality in Returns. The P-Value is zero to six decimal places. This type of distributions are common in finance. In our case because the sample is large enough we can assume that our distributions are not far from normal.

The distribution of returns shows kurtosis “fat tails” but we can safely assume that the distribution is not far from normal. A normal distribution is not skewed and its kurtosis=3. If the P-Value at the bottom is larger than 0.05 we can accept that the returns are normally distributed at the 95% confidence level. In this case the normality has to be rejected mathematically, but it is a common technique to consider such large samples ”normal” in time-series analysis. If the sample was much smaller the assumption of normality would break down.
Fig 18: Histogram of Series Statistics for British Pound Logarithmic Returns. The distribution of returns shows kurtosis "fat tails" but we can safely assume that the distribution is not far from normal.

Fig 19: Histogram of Series Statistics for Euro Logarithmic Returns. The distribution of returns shows kurtosis "fat tails - leptokurtic" but we can assume that the distribution is not far from normal.
Fig 20: Correlogram of Euro Time-Series. Autocorrelation coefficients are considered significant when they are outside the interval $[-1.96 \cdot \sqrt{T}, 1.96 \cdot \sqrt{T}]$, $T=$ Number of Observations. The interval for our case is $[-0.039121834, +0.039121834]$. In this case the Ljung-Box test statistic rejects the null hypothesis of no-autocorrelation even at the 99% confidence level.
Correlogram - Daily Series Data

Fig21: Correlogram of British Pound. In this case the Ljung-Box test statistic rejects the null hypothesis of no-autocorrelation at the 99% confidence level. The series terms show correlation.
Correlogram of Japanese Yen. In this case the Ljung-Box test statistic rejects the null hypothesis of no-autocorrelation at the 99% confidence level for all the 36 lags considered. The series terms show correlation.

Fig22: Correlogram of Japanese Yen. In this case the Ljung-Box test statistic rejects the null hypothesis of no-autocorrelation at the 99% confidence level for all the 36 lags considered. The series terms show correlation.
Stationarity Testing

Fig23a: Dickey-Fuller Test on EURO Values time-series. The Null hypothesis that a unit root exists cannot be rejected because the test-statistic is not more negative than the critical value of the test.
Fig 23b: Dickey-Fuller Test on EURO Values First Differences time-series. The Null hypothesis that a unit root exists can be safely rejected because the test-statistic is more negative than the critical value of the test. The series is stationary.
Fig24a: Dickey-Fuller Test on GBP Values time-series. The Null hypothesis that a unit root exists cannot be rejected because the test-statistic is not more negative than the critical value of the test.
Fig 24b: Dickey-Fuller Test on GBP Values First Differences time-series. The Null hypothesis that a unit root exists can be safely rejected because the test-statistic is more negative than the critical value of the test. The series is stationary.
Fig25a: Dickey-Fuller Test on Yen Values time-series. The Null hypothesis that a unit root exists cannot be rejected because the test-statistic is not more negative than the critical value of the test.
As we saw in the previous graphs the Dickey-Fuller test showed that the data show stationarity. We also tested the time series of Logarithmic returns for the three currencies for stationarity with the Dickey-Fuller test. The Null-Hypothesis that there is a unit root can be safely rejected at the 99% confidence level. The logarithmic returns show stationarity, hence the study of the data is not very complicated.
Chapter 6

Description of the VAR model applied

The VAR model applied in our case is trying to connect three 10 year time-series of daily observations of Euro, GBP, Yen. The calculations are included in the Eviews file VAR_MODEL.wf1. After applying many different ARMA models, and VAR models with various lengths on the data, we found the particular VAR model which better fits the data-sample. The model was tested in the EXCEL file ModelTest.xlsx and was successful for short term predictions. The model was more successful in the study of Euro time-series. The model was tested against the internal random number generator of EXCEL and the probabilities of successful prediction of the movement (upwards or downwards) are shown below:

**Total Sample Average Values:**
- Failure €: 51.24%
- Failure £: 52.152%
- Failure ¥: 50.3%

**For 10 trading days forecast:**
- Failure €: 33.33%
- Failure £: 40.0%
- Failure ¥: 50.0%

**For 5 trading days forecast:**
- Failure €: 25.0%
- Failure £: 60.0%
- Failure ¥: 40.0%

In the calculation of the accuracy of the model, the sets of observations (5,10 trading days) were collected randomly. It is very possible to find other values if somebody tests the model in a different interval.

The model is presented in the following table:
### Vector Autoregression Estimates

**Date:** 09/07/10  **Time:** 21:11  
**Sample (adjusted):** 7,251  
**Included observations:** 2,505 after adjustments  
**Standard errors in () & t-statistics in []**

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Fig 26: The VAR model that better fits the data is a lag order 6.
**Fig 27a:** Most of the tests show order 6 as the best selection.

---

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<th>SC</th>
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* indicates lag order selected by the criterion

LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion
### Correlogram of RESID

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<td>0.004</td>
<td>0.001</td>
<td>48.447</td>
<td>0.090</td>
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</tbody>
</table>

Fig27b: In this case the Ljung-Box test statistic does not reject the null hypothesis of no-autocorrelation at the 99% confidence level. The Residuals are not correlated at the 99% confidence level. The model fits the data properly when the residuals do not show autocorrelation.
VAR Granger Causality - Daily Series Data

![VAR Granger Causality Test results]

Fig 28: We reject Null Hypothesis that GBP and Yen don’t cause movements in Euro. Euro is likely to cause movements in GBP, but not Yen. (No Rejection of Null Hypothesis at the 99% confidence level). Also Yen is not moving with GBP and Euro. (No Rejection of Null Hypothesis at the 99% confidence level). These are the results of the Granger Causality Test.
VAR Method - Monthly Series Data

We applied the method described before in a sample of monthly FX Rates. The model was in this case more successful showing that monthly prices enclose more information about the market. The model was applied in €, £, ¥ for a period of more than ten years (2000-2010). Usually VAR models are used in weekly or monthly data, but this does not mean that these models cannot be effective on daily data. Monthly data or quarterly data can be combined with other macroeconomic factors like inflation, unemployment, interest rates. This is the reason for their popularity especially in the academic research.

<table>
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<th>Average Failure %</th>
<th>€</th>
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<th>¥</th>
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</thead>
<tbody>
<tr>
<td>Failure %</td>
<td>35.71428571</td>
<td>46.03174603</td>
<td>47.61904762</td>
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</tbody>
</table>

The results are presented in the following graphs:
Fig 30: The Model Results for Euro approach the Real FX Rate.

Fig 31: The Model Results for GBP approach the Real FX Rate.
VAR Method - $\$/¥ Monthly Series Data

Fig32: The Model Results for Yen approach the Real FX Rate.
VAR Model - € £ ¥ Monthly Series Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RXISUS_N_M_EU</th>
<th>RXISUS_N_M_UK_GBP</th>
<th>_JP_YEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>RXISUS_N_M_EU(-1)</td>
<td>1.467464646591614</td>
<td>0.61518473440473</td>
<td>0.00041433565856356</td>
</tr>
<tr>
<td>RXISUS_N_M_EU(-2)</td>
<td>-0.484230592535090</td>
<td>-0.59571397400636</td>
<td>0.00001560723516708</td>
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<tr>
<td>RXISUS_N_M_UK_GBP(-1)</td>
<td>-0.187082874719436</td>
<td>0.968846281222632</td>
<td>-0.00111871557015043</td>
</tr>
<tr>
<td>RXISUS_N_M_UK_GBP(-2)</td>
<td>0.153793067469014</td>
<td>-0.0143316835612754</td>
<td>0.00070196464656999</td>
</tr>
<tr>
<td>_JP_YEN(-1)</td>
<td>0.10170336013408</td>
<td>0.122058559292899</td>
<td>0.00070195600004113</td>
</tr>
<tr>
<td>_JP_YEN(-2)</td>
<td>-0.18291426010274</td>
<td>13.3606443513102</td>
<td>1.1404893052209</td>
</tr>
<tr>
<td>_JP_YEN(-3)</td>
<td>13.60189765441419</td>
<td>28.20680596529605</td>
<td>0.00070195600004113</td>
</tr>
</tbody>
</table>

R-squared: 0.978765793918032, Adj. R-squared: 0.977704083613933, Sum sq. resid.: 0.10754832500355, S.E. equation: 0.029697202692092, F-statistic: 921.87853906384

Fig33: The Model Results show high $R^2$ and approach the Real FX Rate.
Chapter 7

Geometric Brownian Stochastic Model

Exchange rates often change dramatically on a daily basis for reasons that apparently have little connection to economic and financial variables. Often FX Rates move in the opposite direction of differences in short-term interest rates and PPP across countries. Hence despite being a-theoretical, a stochastic model remains appealing because sometimes it leads to smaller forecasting errors than most other exchange rate models.

In this part of our work a Stochastic Model was applied. A pure Geometric Brownian stochastic process was implemented in MATLAB in order to explain the observations. The file “Random-Process.m” includes the MATLAB code written for this task; The code is included also in the appendix. MATLAB was used because it is more powerful than MS EXCEL and can handle very large arrays of random numbers (size = 400x500000x8 byte = 1525.87890625 Mbyte). We have to note that the stochastic model was very successful for the periods tested.

A geometric Brownian motion is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion (Wiener process). If logarithmic returns show no autocorrelation, then a stochastic model can be very accurate in many cases.

Geometric Brownian motion is expressed mathematically by the following stochastic differential equation, which can be solved analytically when \( \mu \) (the percentage drift) and \( \sigma \) (the percentage volatility) are constants.

\[
\frac{dS_t}{S_t} = \mu dt + \sigma dW_t
\]

\( W_t \) is a stochastic Wiener process, a standard gaussian with zero mean and unit standard deviation. In our case \( S_t = \frac{\$}{T} \) exchange rate. Now if we assume that the initial value of \( S(t) \) is \( S_0 \) then the solution of this stochastic differential equation is:

\[
S_t = S_0 \cdot e^{(\mu - \sigma^2/2)t + \sigma W_t}
\]

The solution is lognormally distributed. It’s expected value is:

\[
E(S_t) = S_0 e^{\mu t}.
\]

For our work, we assumed that for small time intervals of a few days, volatility \( \sigma \) and percentage drift \( \mu \) remain unchanged. We assume also that there are real risk free rates in the markets we examined. The returns follow a Normal distribution (The underlying is a Log-normal distribution - Efficient-markets). There are no transaction costs or taxes, there are no restrictions on short selling, and no arbitrage.

Assuming now that an investor in currency earns risk-less profits by just keeping money in a bank account, we have that after a period \( t \) his money will be \( S_t = S_0 e^{rt} \). Hence the previous equation becomes:

\[
S_t = S_0 \cdot e^{rt + (\mu - \sigma^2/2)t + \sigma W_t}
\]

If the investor was holding his capital in US$ he would earn \( r_a \) risk-free interest rate, hence the
net gain he earns by keeping his money in British pounds will be \( r - r_a \).

\[
S_t = S_0 \cdot \exp((r - r_a) t + (\mu - \sigma^2 / 2) t + \sigma W_t), \quad r : \text{British interest rate}, \quad r_a : \text{US interest rate}
\]

and \( S_t = \frac{\$}{\£} \) exchange rate.\(^1\)

The major weakness of the Method is that the potential investor has to make a very serious choice about the volatility and the future drift (positive or negative). Historical values of volatility and drift can be calculated easily from the series. A Foreign Exchange investor can assume that for a small time horizon these values can be taken from the series during the last semester. The choice of the drift is based on the information and finally on the expectations of the investor. The graph Fig30d shows that in the **long run, the total drift is about the difference in the interest rates**, \( drift = r - r_a + \mu, \mu = 0; \) this is expected theoretically. In our case the total drift of the Geometric Brownian Motion is \( r - r_a \), hence the assumption of the interest rate parity holds in the long run. Fig30b shows the volatility of the sample. The last period the volatilities rose and this affects the quality of the forecast. For long term forecasting (1 year or more) the random walk can be very accurate if the investor calculates the difference in interest rates each time. Interest rates change often several times throughout the year. For example during 2008 according to this model we expect when :

\[
\begin{align*}
& r_{\text{Britain}} - r_{\text{US}} < 0 \quad \text{then } \$/\£ \text{ will fall (\£ will depreciate)} \\
& r_{\text{EU}} - r_{\text{US}} < 0 \quad \text{then } \$/\€ \text{ will fall (\€ will depreciate)} \\
& r_{\text{Japan}} - r_{\text{US}} < 0 \quad \text{then } \$/\¥ \text{ will fall and finally ¥/$ \text{ is expected to rise (¥ will depreciate)}}
\end{align*}
\]

We will see later that euro and pound followed the expectations of the model, and lost value against dollar. Yen moved in the opposite direction, instead it finally gained value against the American currency. And this shows that it is not only the interest rates that affect the exchange rates but exchange rate forecasting is a much more complicated process. Japan is an industrial nation, with very high exporting activity and relatively low imports from USA. It is quite possible that this is the reason that yen appreciated since 1961 almost every year against the dollar.

Many support the idea of computing the average values of volatility and drift during the last week. According to my opinion another idea is to calculate the average value of volatility the last month (20 trading days). The drift can be found with trial and error for a specific time horizon, from the previous values of the sample. In other words for short time horizons an investor can use the average value of volatility during the last few observations, and then try different drift rates in the program till the calculated values are near the observed exchange rates. Usually the value for the drift cannot be more than 5% or less than -5%. We took values of volatility from the graph Fig30b, and drifts based on the trial and error method. The other factors of the model like the risk-free rates were found from the FED, ECB and bank of Japan for the specific period.

It is quite clear that the interest rate parity along with the Black-Scholes model hold quite well in the data. In the short run many other factors seem to play a role in the FX rates. Short term forecasting is more difficult with this method because fluctuations occur in the drift rate during a period of about 10 trading days. Information is the most important thing in order to decide about the following strategy. Usually large market makers in FX have an information advantage because they can track the accounts of their clients, and see the offer and demand for a specific currency earlier. This information asymmetry can drive to market manipulation by large investment banks.

In the following graphs we show the results of the simulation in \( \£, \¥, \text{and } \$ \). We assumed in some of the simulations that the drift was \( r - r_a, \mu = 0 \). In other forecasts the rate \( \mu \) used is mentioned in the graphs. It is clearly visible that short term forecasting is affected by the right value of the drift.

\(^1\)Options Futures & Other Derivatives prof John Hull
Historical Volatility

Fig 30b: Historical Volatility. Note that the pound is very volatile during the last period (just before our forecast)
Historical Drifts for €&£ Last 100 Days

€ Blue Line & £ Red Line Last 100 Days

Fig30c1: Historical Drifts Pound - Red and Euro - Blue
Historical Drifts ¥ Last 100 Days

Fig30c2: Historical Drifts YEN
Historical Drifts for €&£ Last Year (252 Days)

€ Blue Line & £ Red Line Last 252 Days

Fig30d: Historical Drifts Pound - Red and Euro - Blue Last 252 Days
MATLAB Simulation Euro 12/22/2008 - 12/30/2008 (5 trading days)

Sample Variance = 0.0025  
Mean Value of the Future FX price Distribution : 1.394122088  
Median Value of the Future FX price Distribution : 1.393122660  
Real Value= 1.4085 $

Fig30E1: Simulation for Euro in the interval 12/22/2008 to 12/30/2008 (5 trading days)

\[ \mu = 0.02318206 \]

MATLAB Simulation Euro 12/22/2008 - 12/30/2008 (5 trading days) Drift

Sample Variance = 7.7241e-004  
Mean Value of the Future FX price Distribution : 1.407061349  
Median Value of the Future FX price Distribution : 1.406775231  
Real Value= 1.4085 $

Fig30E2: Simulation for Euro in the interval 12/22/2008 to 12/30/2008 (5 trading days)
MATLAB Simulation Euro 12/04/2008 - 12/18/2008 (10 trading days)

Sample Variance = 0.0042
Mean Value of the Future FX price Distribution : 1.28232167
Median Value of the Future FX price Distribution : 1.28022564
Real Value = 1.4298 $

Fig31: Simulation for Euro in the interval 12/04/2008 to 12/18/2008 (10 trading days)

MATLAB Simulation Euro 12/04/2008 - 12/18/2008 (10 trading days) Drift $\mu = 0.13563485$

Sample Variance = 0.0047
Mean Value of the Future FX price Distribution : 1.430747118
Median Value of the Future FX price Distribution : 1.428760432
Real Value = 1.4298 $

Fig31b: Simulation for Euro in the interval 12/04/2008 to 12/18/2008 (10 trading days)
MATLAB Simulation Euro 11/18/2008 - 12/31/2008 (30 trading days)

Sample Variance = 0.0123
Mean Value of the Future FX price Distribution : 1.263323037
Median Value of the Future FX price Distribution : 1.257499602
Real Value = 1.3919$

Fig32: Simulation for Euro in the interval 11/18/2008 to 12/31/2008 (30 trading days)

MATLAB Simulation Euro 01/02/2008 - 12/31/2008 (252 trading days)

Sample Variance = 0.0978
Mean Value of the Future FX price Distribution : 1.422275932
Median Value of the Future FX price Distribution : 1.382535931
Real Value = 1.3919$

Fig32b: Simulation for Euro in the interval 01/02/2008 - 12/31/2008 (252 trading days). Note how accurate the forecast is for one year time horizon. In this case drift = r - r_a + \mu, \mu = 0
MATLAB Simulation GBP 12/22/2008 - 12/30/2008 (5 trading days)

Sample Variance = 0.0046  
Mean Value of the Future FX price Distribution : 1.479398576  
Median Value of the Future FX price Distribution : 1.477491487  
Real Value= 1.4395$

Fig33: Simulation for GBP in the interval 12/22/2008 to 12/30/2008 (5 trading days)

MATLAB Simulation GBP 12/04/2008 - 12/18/2008 (10 trading days)

Sample Variance = 0.0114  
Mean Value of the Future FX price Distribution : 1.476966865  
Median Value of the Future FX price Distribution : 1.472222120  
Real Value= 1.5103$

Fig34: Simulation for GBP in the interval 12/04/2008 - 12/18/2008 (10 trading days)
MATLAB Simulation GBP 12/04/2008 - 12/18/2008 (10 trading days) Drift $\mu = +0.028$

Sample Variance = 0.0054
Mean Value of the Future FX price Distribution : 1.510346644
Median Value of the Future FX price Distribution : 1.508334910
Real Value= 1.5103$

Fig34b: Simulation for GBP in the interval 12/04/2008 - 12/18/2008 (10 trading days) Drift=+0.028

MATLAB Simulation GBP 11/18/2008 - 12/31/2008 (30 trading days)

Sample Variance = 0.0355
Mean Value of the Future FX price Distribution : 1.497419351
Median Value of the Future FX price Distribution : 1.483238677
Real Value= 1.4619$

Fig35: Simulation for GBP in the interval 11/18/2008 - 12/31/2008 (30 trading days)
MATLAB Simulation GBP 11/18/2008 - 12/31/2008 (30 trading days) Drift $\mu = -0.01$

Sample Variance = 0.0147  
Mean Value of the Future FX price Distribution : 1.462770071  
Median Value of the Future FX price Distribution : 1.456537521  
Real Value= 1.4619$

![Histogram of GBP prices](image1.png)

Fig35b: Simulation for GBP in the interval 11/18/2008 - 12/31/2008 (30 trading days) Drift=-0.01

MATLAB Simulation GBP 01/02/2008 - 12/31/2008 (252 trading days) Drift $\mu = -0.0122$

drift = -0.0122  
r = -0.0140  
Sample Variance = 0.2827  
Mean Value of the Future FX price Distribution : 1.510009398  
Median Value of the Future FX price Distribution : 1.410043763  
Average Value of the Mean and Median : 1.460026580  
Real Value= 1.4619$

![Histogram of GBP prices](image2.png)

Fig35c: Simulation for GBP in the interval 01/02/2008 - 12/31/2008 (252 trading days) Drift=-0.0122
MATLAB Simulation YEN 12/22/2008 - 12/30/2008 (5 trading days)

Sample Variance = 4.0809
Mean Value of the Future FX price Distribution : 90.674919226
Median Value of the Future FX price Distribution : 90.646157238
Real Value= 90.37 ¥/$

Fig36: Simulation for YEN in the interval 12/22/2008 to 12/30/2008 (5 trading days)

MATLAB Simulation YEN 12/04/2008 - 12/18/2008 (10 trading days)

Sample Variance = 8.2666
Mean Value of the Future FX price Distribution : 91.559638552
Median Value of the Future FX price Distribution : 91.502956246
Real Value= 89.81 ¥/$

Fig37: Simulation for YEN in the interval 12/04/2008 - 12/18/2008 (10 trading days)
MATLAB Simulation YEN 11/18/2008 - 12/31/2008 (30 trading days)

Sample Variance = 25.4786
Mean Value of the Future FX price Distribution : 93.584824366
Median Value of the Future FX price Distribution : 93.419058257
Real Value= 90.79 ¥/$

Fig38: Simulation for YEN in the interval 11/18/2008 - 12/31/2008 (30 trading days)

MATLAB Simulation YEN 11/18/2008 - 12/31/2008 (30 trading days) Drift $\mu = -0.05$

Sample Variance = 23.1831
Mean Value of the Future FX price Distribution : 90.081750341
Median Value of the Future FX price Distribution : 89.934073733
Real Value= 90.79 ¥/$

Fig39: Simulation for YEN in the interval 11/18/2008 - 12/31/2008 (30 trading days) Drift = -0.05
MATLAB Simulation YEN 12/04/2008 - 12/18/2008 (10 trading days) Drift $\mu = -0.05$

Sample Variance = 7.9472
Mean Value of the Future FX price Distribution : 90.374947153
Median Value of the Future FX price Distribution : 90.318532155
Real Value= 89.81 ¥/$

![Fig40a: Simulation for YEN in the interval 12/04/2008 - 12/18/2008 (10 trading days) Drift = -0.05](image)

MATLAB Simulation YEN 01/02/2008 - 12/31/2008 (252 trading days) Drift $\mu = -0.0258$

Sample Variance = 210.0966
Mean Value of the Future FX price Distribution : 92.121379532
Median Value of the Future FX price Distribution : 90.734088841
Average Value of the Mean and Median : 91.427734186
Real Value= 89.81 ¥/$

![Fig40b: Simulation for YEN in the interval 01/02/2008 - 12/31/2008 (252 trading days) Drift = -0.0258](image)
Chapter 8

Summary

In this data sample, our analysis clearly proved, that the efficiency of the Markets hypothesis holds well, the Logarithmic Returns show no autocorrelation and their distributions are not very far from normal. As we saw before, \(\text{€-£}\) series show high positive correlation and \(\text{¥-€, ¥-£}\) show much lower correlation. For smaller sub-periods, yen moved even in the opposite direction. An investor can take advantage of this characteristic in order to diversify the risk he takes by keeping euros and pounds in his portfolio.

From our results, it is visible that the most effective strategy in FX trading is the combination of a random model, based on solid theoretical assumptions which are based on historical data. A well informed risk averse investor, can succeed in his forecast, if he combines a model like the one presented before, with a good knowledge about the macroeconomic variables involved in FX. It is clear that the choice of drift and volatility is very important in order the random model to be accurate. Historical values of the macroeconomic variables along with today’s information about them are very important in determining the exchange rate direction (drift \(\mu\)). Future interest rate uncertainty affects the exchange rate between two countries. Consequently exchange rate risk is connected with interest rate risk.

Another important result obtained from our analysis, is that the use of very old data can distort the final outcome. Important is the fact that relative PPP holds in FX, but one has to consider the fact that large deviations occurred for large periods. Hence most of the times, an investor cannot use PPP criterion alone to define the movement of a currency over a short time interval. We saw that overvalued currencies usually fall, and follow the PPP rule, with a lag. Pound/Dollar exchange rate does not follow PPP as we have seen before, but that does not mean that the pound is the rule. Yen observed value is higher than the PPP value after 1995, hence many expect that finally yen’s value will fall against the U.S. dollar. The weaknesses observed in the Japanese economy and the constant growth of the other Asian economies during the last decade may reduce the value of the yen. Other analysts believe that the US dollar won’t manage to keep its today’s position as the benchmark currency in global trading. Consequently in the long run dollar will depreciate even more.

Exchange rates fluctuate often almost rapidly, but in the long run they have to follow specific macroeconomic rules. Deviation from these rules like the Relative PPP, and the interest rate parity, usually cannot last forever. In the long run the exchange rate will reach equilibrium around the value imposed by the macroeconomic factors that characterize the economies of the countries. Hence the FX rate direction can be predicted if the interest rates do not change sharply, and the countries are trustworthy in their announcements and actions. For shorter time intervals, the combination of a mixed model like VARs, with a stochastic model, can be very effective. Most of the Investment Banks use models that combine both, and many times even behavioral factors.
Chapter 9

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Chapter 10

Appendix

MATLAB code for RandomProcess.m

% Clean variables and screen
clc

clear all

% Initial Prices for the calculation
S0 = 109.58; sigma = 0.045; r = 0.5/100; n = 500000;
Dt = 252*1/252; m = 40; ra = 2.18/100;
dif = r-ra

drift = -0.009
r = dif + drift

% Random Paths for the FX Price
Spath = S0*[ones(n,1),cumprod(exp((r-0.5*sigma^2)*Dt+sigma*sqrt(Dt)*randn(n,m)),2)];

% Monte Carlo Method for arithmetic average
arithave = mean(Spath,2);
SampleVariance = var(arithave)
MeanValue = mean(arithave);
MedianValue = median(arithave);
disp('Mean Value of the Future FX price Distribution :');
fprintf('%22.9f
',MeanValue);
disp('Median Value of the Future FX price Distribution :');
fprintf('%22.9f
',MedianValue);

AvgValue=(MeanValue+MedianValue)/2;
disp('Average Value of the Mean and Median :');
fprintf('
%22.9f
', AvgValue);
[count, BinCenter] = hist(arithave, 30);
figure
bar(BinCenter, count/sum(count), 1, 'r')
xlabel('Terminal FX Price')
ylabel('Probability')
title('Lognormal Terminal FX Prices')
Fig30: Japanese risk free rate (Average Monthly Values) : Source Bank of Japan

Fig31: UK interest rate graph : Source Bank of England

Official Bank of England Rate
2006 3 Aug 4.7500 9 Nov 5.0000
2007 11 Jan 5.2500 10 May 5.5000 5 July 5.7500 6 Dec 5.5000
2008 7 Feb 5.2500 10 April 5.0000 8 Oct 4.5000 6 Nov 3.0000 4 Dec 2.0000
2009 8 Jan 1.5000 5 Feb 1.0000 5 Mar 0.5000
### Key ECB Interest Rates

The Governing Council of the ECB sets the key interest rates for the euro area.

#### Data

(Interest rate levels in percentages per annum)

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<th>Date</th>
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<td>Variable rate tenders</td>
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Fig32: ECB interest rates Source: European Central Bank
US 6 Month CDs
01/2008, 3.71
02/2008, 2.99
03/2008, 2.70
04/2008, 2.86
05/2008, 2.84
06/2008, 3.09
07/2008, 3.13
08/2008, 3.11
09/2008, 3.82
10/2008, 4.37
11/2008, 2.83
12/2008, 2.18
01/2009, 1.53
02/2009, 1.75

Purchasing Power Bloomberg
### Big Mac Index

**How Far from Fair Value is Your Currency?**

The Economist's Big Mac Index ([July 22nd, 2010 Big Mac prices](https://www.economist.com)) valued at today's exchange rates.

<table>
<thead>
<tr>
<th>Country</th>
<th>Big Mac Price</th>
<th>Implied PPP rate + Today's Exchange Rate ($) = Valuation against the USD, % ++</th>
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<tbody>
<tr>
<td>United States</td>
<td>$ 3.73</td>
<td>3.7300 1.0000 1.0000</td>
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<tr>
<td>Argentina</td>
<td>Peso 14</td>
<td>3.4960 3.75 4.0046 -6.3577</td>
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<tr>
<td>Australia</td>
<td>A$ 4.35</td>
<td>4.0711 1.17 1.0685 9.4993</td>
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<tr>
<td>Brazil</td>
<td>Real 8.71</td>
<td>4.9610 2.33 1.7557 32.7106</td>
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<td>Britain</td>
<td>£ 2.29</td>
<td>3.5787 0.61 0.6399 -6.726</td>
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<tr>
<td>Canada</td>
<td>CS 4.17</td>
<td>4.0341 1.12 1.0337 8.3487</td>
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<tr>
<td>Chile</td>
<td>Peso 1750</td>
<td>3.4364 469 509.250 -7.9038</td>
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<td>Yuan 13.2</td>
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<td>Colones 2000</td>
<td>3.8977 536 513.120 4.4590</td>
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<td>Czech Republic</td>
<td>Koruna 67.6</td>
<td>3.5688 18.1 19.8417 -4.4436</td>
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<td>Denmark</td>
<td>DK 28.5</td>
<td>4.9900 7.63 5.7114 33.5925</td>
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<td>Pound 13</td>
<td>2.2249 3.48 5.8429 -40.4405</td>
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<td>Kroon 32</td>
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<td>€ 3.38</td>
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<td>7.3490 12.1 6.1233 97.6059</td>
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<td>Pakistan</td>
<td>Rupee 210</td>
<td>2.4064 56.30 87.2660 -35.4846</td>
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