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1.2.1. Measuring market risk
Abstract
Market risk is normally associated with instruments traded on well defined markets, though increasingly, techniques are used to assess the risk arising from over the counter instruments, and/or traded items where the market is not very liquid. The value of any instrument will be a function of price, coupon, coupon frequency, time, interest rate and other factors. If a bank is holding instruments on account (for example equities, bonds), then it is exposed to market risk, the risk that the price of the instrument will be volatile. Systematic market risk is caused by a movement in the prices of all market instruments because of, for example, a change in economic policy. Unsystematic market risk arises in situations where the price of one instrument moves out of line with other similar instruments, because of an event related to the issuer of the instrument.

Value at risk (VaR) has become the standard measure that financial analysts use to quantify market risk. VaR is defined as the maximum potential change in value of a
portfolio of financial instruments with a given probability over a certain horizon. More specific, it is the maximum loss which can occur with X% confidence over a holding period of n days. The results produced by a VaR model are simple for all levels of staff from all areas of an organization to understand and appreciate. That is why VaR has been adopted so rapidly.

Increased volatility of financial markets and rapid enhancement of computer systems gave banks an important characteristic. Bank would now be a risk manager compared to their former “traditional” role. The science of risk management is not a mature field of knowledge but it constantly evolves. The most prominent of risks – on which a bank is exposed – is market risk since it reflects the potential economic loss caused by the decrease in the market value of a portfolio. Value at Risk (VaR) is the most common measure that financial analysts, banks and supervisors use to measure market risk.

The concept of systematic risk has played an important role in finance since Markowitz formalized the notion that investors should hold a diversified portfolio under uncertainty. There have been lots of financial studies concerning beta coefficient: its estimation has “traditionally” been achieved by running a market model regression. Is it important for a company and the investor too? What about the stability of beta and its behavior in bull and bear markets? May the downside risk, as measured by the beta corresponding to bear market, be an appropriate measure of portfolio risk? All of these questions and more other will be answered, in order to be clarified, after reading the paper, what market risk is, which are the specific risks that modern banks have to measure, what Value at Risk is and how we can use it to measure all of these risks, what other types of risk measurement exists, what beta coefficient is and what systematic risk is.

On this project the VaR approach and the most popular coefficient in finance, beta, will be presented while trying to analyze market risk, on which a bank is exposed. Beta coefficient of five Greek banks’ stocks will be defined in bull and bear markets using Ordinary Least Squares regression (OLS). The VaR of a hypothetical portfolio will be also estimated using “beta model” and will be compared to the variance – covariance method.

**Introduction**

In finance there is the opinion that prices are not only affected by external facts but and from causes that are related to the internal operation of the markets and the psychology of the investors. Theoretically, if someone is in order to know the cause of a fact, then he can predict, manage and control his risk. Many researches, that were made the last decades, have led to the conclusion that the volatility of the prices could not be explained by the known financial theories and could not be depicted in normal distribution diagrams. From 1916 until 2003 variations above 3,4% of Dow Jones were observed for more than 1000 days, which normally could have never been above 58 days. In 2002 Shiller and other scholars came to the conclusion that such disorders come in conflict with efficient markets. Many thought that all this instability was due to the change of pricing model. For a long time the known Merrill Lynch was using the
Capital Asset Pricing Model (CAPM), publishing catalogues with rates of systematic risk that were useful to brokers and investors in order to build efficient portfolios. However, the recession of 19/10/1987 made Dow Jones to fall by 29,2% and caused the collapse of all these well designed portfolios and everyone then could recognize the complexity of the world market economy.

It is a fact that the increased volatility of the variance, that is observed longitudinal, even if important financial information are missing, makes the effective risk management to be one of the most important and crucial section for the financial researchers and for the modern banks. Every financial institution should in daily basis evaluate the risk value and hold their capital for protection from contingent losses.

A financial institution is supposed to be the basic risk manager and one general procedure that measure the market risk that its portfolio is exposed; it is the Value at Risk (VAR). The volatility of each stock in a portfolio depends on the correlation with the market and that is defined by the beta or systematic risk.

But how is beta acting when the market moves upwards or downwards and how the volatility of beta can influence the Value at Risk of the portfolio? This research is supposed to answer this question by using the beta model combined with a research of Kim and Zumwalt on instability of systematic risk in bull and bear markets.

The first chapter shows how the crisis in America influences the global economy and the European markets and the changing of the role of the financial institutions to the new reality and at last the importance of risk management of the portfolios. Also, in this chapter it is shown how we estimate those risks concentrating mainly in Value at Risk method for measuring market risk.

The second chapter has a brief presentation of investing theories from the era of Markowitz and the characteristics of systematic risk are being analyzed. Furthermore, it is shown the attitude of the investors towards risk.

The empirical part in chapter three shows the volatility of beta of five Greek bank stocks in bull and bear markets and an estimation of Value at Risk of a hypothetical portfolio is been included by using the variance and beta model in comparison with the covariance.

CHAPTER 1: Market risk in modern banks and VaR estimation

1.1. Modern bank and financial risks

1.1.1. A crisis of historic proportions

The financial crisis that hit the global economy since the summer of 2007 is without precedent in post-war economy history. Although its size and extent are exceptional, the crisis has many features in common with similar financial-stress driven recession episodes in the past. The crisis was preceded by long period of rapid credit growth, low risk premiums, abundant availability of liquidity, strong leveraging, soaring asset prices and the development of bubbles in the real estate sector. Over-stretched leveraging positions rendered financial institutions extremely vulnerable to corrections
in asset markets. As a result a turn-around in a relatively small corner of the financial system was sufficient to topple the whole structure. Such episodes have happened before (e.g. Japan and the Nordic countries in the early 1990s, the Asian crisis in the late 1990s). However, this time was different, with the crisis being global akin to the events that triggered the Great Depression of the 1930s.

While it may be appropriate to consider the Great Depression as the best benchmark in terms of its financial triggers, it has also served as a great lesson. At present, governments and central banks are well aware of the need to avoid the policy mistakes that were common at the time, both in the Europe and elsewhere. Large-scale bank runs have been avoided, monetary policy has been eased aggressively, and governments have released substantial fiscal stimulus. Unlike the experience during the Great Depression, countries in Europe and elsewhere have not resorted to protectionism at the scale of 1930s. It demonstrates the importance of Europe coordination, even if this crisis provides an opportunity for further progress in this regard.

In its early stages, the crisis manifested itself as an acute liquidity shortage among financial institutions as they experienced ever stiffer market conditions for rolling over their debt. In this phase, concerns over the solvency of financial institutions were increasing, but a systematic collapse was deemed unlikely. This perception dramatically changed when a major investment bank (Lehman Brothers) defaulted in September 2008. Confidence collapsed, investors massively liquidated their positions and stock markets went into a tailspin. From then onward the Europe economy entered the steepest downturn on record since the 1930s. The transmission of financial distress to the real economy evolved at record speed, with credit restraint and sagging confidence hitting business investment and household demand, notably for consumer durables and housing. The cross-border transmission was also extremely rapid, due to the tight connections within the financial system itself and also the strongly integrated supply chains in global product markets.

The ongoing recession is thus likely to leave deep and long-lasting traces on economic performance and entail social hardship of many kinds. Job losses can be contained for some time by flexible unemployment benefit arrangements, but eventually the impact of rapidly rising unemployment will be felt, with downturns in housing markets occurring simultaneously affecting households. The fiscal positions of governments will continue to deteriorate, not only for cyclical reasons, but also in a structural manner as tax bases shrink on a permanent basis and contingent liabilities of governments stemming from bank rescues may materialize. An open question is whether the crisis will weaken the incentives for structural reform and thereby adversely affect potential growth further, or whether it will provide an opportunity to undertake far-reaching policy actions.

1.1.2. Crisis management

The current crisis has demonstrated the importance of a coordinated framework for crisis management. It should contain the following building blocks:
• **Crisis Prevention** to prevent a repeat in the future. This should be mapped onto a collective judgment as to what the principal causes of the crisis were and how changes in macroeconomic, regulatory and supervisory policy frameworks could help prevent their recurrence. Policies to boost potential economic growth and competitiveness could also bolster the resilience to future crisis.

• **Crisis Control and Mitigation** to minimize the damage by preventing systemic defaults or by containing the output loss and easing the social hardship stemming from recession. Its main objective is thus to stabilize the financial system and the real economy in the short run. It must be coordinated across the Europe in order to strike the right balance between national preoccupations and spillover effects affecting other Member States.

• **Crisis Resolution** to bring crises to a lasting close, and at the lowest possible cost for the taxpayer while containing systemic risk and securing consumer protection. This requires reversing temporary support measures as well action to restore economies to sustainable growth and fiscal paths. Inter alia, this includes policies to restore banks’ balance sheets, the restructuring of the sector and an orderly policy exit.

The bank crisis and the bankruptcy of financial institutions in the last two decades had negative causes in real economy and especially for the grow countries. In those countries the liquidity in the markets comes from banks, which in their portfolios keep a great percentage of the government debt. The high movement of capital in 1990s led to the dependence in between economies all over the world and it was shown that a crisis in only one market could influence and have negative reactions and to other countries too.

1.1.3. **Developments in financial sector**

The causes of the upper crisis were crucial for the expectations of the investors, the savers and the businesses, that have shown prefer in liquidity. The business capital was redistributed. Also the high variability of interest rates increased the demand of new, innovative and complicated financial instruments and played a role in the enlargement of the banking sector. Banks started to transfer the risk to the lenders and the borrowers by reducing the period of loans and adapting floating interest rates.

Risk management, after variation appeared seems to be one of the most important sectors for the financial institutions and banks. The globalization of the markets and the growth of technology and management information systems contributed in order bank not only to be gatherer, distributor of financial information and to have the role of intermediary but to actual be the risk manager.

In past the main strategy of a financial institution was to manipulate the assets with purpose:

- The biggest profit in loan portfolios with the lowest risk
- Right predictions in order to have liquidity
So the liquidity came from deposits that had standard interest rates, which meant that banks among each other were no competitive. In our days the competitive environment make the banks to have a goal of maximizing the profits and in order to do so it is needed first of all to manage their own risk, giving basis in assets, where the quality evaluation of bank loan portfolio and market risk is being measured.

1.1.4. Why is risk management needed?
Recent financial disasters in financial and non-financial firms and in governmental agencies point up the need for various forms of risk management. Financial misadventures are hardly a new phenomenon, but the rapidity with which economic entities can get into trouble is. Banks and similar financial institutions need to meet forthcoming regulatory requirements for risk measurement and capital. However, it is a serious error to think that meeting regulatory requirements is the sole or even the most important reason for establishing a sound, scientific risk management system. Managers need reliable risk measures to direct capital to activities with the best risk/reward ratios. They need estimates of the size of potential losses to monitor positions and create incentives for prudent risk-taking by divisions and individuals.

Risk management is the process by which managers satisfy these needs by identifying key risks, obtaining consistent, understandable, operational risk measures, choosing which risks to reduce and which to increase and by what means, and establishing procedures to monitor the resulting risk position.

Risk, may be defined as reductions in firm value due to changes in the business environment. Typically, the major sources of value loss are identified as:

- **Market Risk** is the change in net asset value due to changes in underlying economic factors such as interest rates, exchange rates and equity and commodity prices.

- **Credit Risk** is change in net asset value due to changes the perceived ability of counterparties to meet their contractual obligations.

- **Operational Risk** results from costs incurred through mistakes made in carrying out transactions such as settlement failures, failures to meet regulatory requirements and untimely collections.

- **Performance Risk** encompasses losses resulting from the failure to properly monitor employees or to use appropriate methods.

1.1.5. Bank portfolio
Portfolio is a collection of investments all owned by the same individual or organization. These investments often include stocks, which are investments in individual businesses; bonds, which are investments in debt that are designed to earn
interest; and mutual funds, which are essentially pools of money from many investors that are invested by professionals or according to indices.

Prudence suggests that investors should construct an investment portfolio in accordance with risk tolerance and investing objectives. For example, a conservative investor might favor a portfolio with large cap value stocks, broad-based market index funds, investment-grade bonds and a position in liquid, high-grade cash equivalents. In contrast, a risk loving investor might add some small cap growth stocks to an aggressive, large cap growth stock position, assume some high-yield bond exposure, and look to real estate, international and alternative investment opportunities for his or her portfolio.

The bank's object must be primarily that of insuring its own liquidity and solvency at the same time that it distributes its funds fairly and equitably among the different elements in its clientele. First of all, it needs to analyze its liabilities in a careful and scientific manner. We may assume that the bank has, let us say, $250,000 of capital and that it has succeeded in developing a deposit line of, say, $750,000. Analysis shows that of this deposit line $250,000 is in time deposits or certificates of deposit which move very slowly and are practically continuously renewed. This leaves $500,000 of demand deposits, and a study of them makes it reasonably sure that there will be a rapid turnover only, say, in the spring and again in the autumn. The bank is therefore in position first of all to invest its funds in assets of fairly long and non-liquid character corresponding to its $250,000 of inactive deposits. Perhaps it may carry a part of this amount in real-estate mortgages or it may think well to invest a portion in sound bonds or government obligations. It thus has as the basis of its portfolio some long-term securities, part of which (the real-estate securities) will not mature for a great while and hence are not liquid, while other portions (government securities) will not mature for a long time, but are very salable and hence may be realized in case of necessity.

Behind the demand-deposit line the banker probably has an approximately equal amount of current notes and other relatively short-term obligations. These may run from a few days up to six months or more, and it should be the effort of the banker first of all to arrange their maturities in such a way as to have them fall due steadily and successively, so as to provide him with the cash he needs to meet the current drafts upon him. For instance, if he has found that during the months of March and April of each year, and again during the months of September and October, he is obliged to make very heavy outlays, his depositors drawing on him and reducing their deposits correspondingly in order that they may liquidate indebtedness for goods, or may transfer their funds to other places, he evidently needs to have maturities of fully equal
amount fall due at about that time. This is for the purpose of providing him with cash in order that he need not reduce his reserve below its normal or average level. His portfolio may, therefore, be conceived of as consisting of fairly long-term loans which, however, are unquestionably payable at maturity and which have been "bunched" so that their maturities will fall due in such a way as to meet the demands which are brought to bear upon the banker at the "peak" periods. The banker, however, has to reckon upon a regular steady flow of funds out of the bank. He is providing cash for the community, and, while he expects about an equal amount of income, he cannot be absolutely sure of it. He will therefore endeavor to carry enough paper falling due from day to day or from week to week to enable him, if he finds it necessary, to reduce his portfolio by failing to make new loans or refusing to renew old ones, and thus get in the cash which he needs to meet the regularly recurring demands to which reference has just been made. The banker, however, will not have been in business very long before he will find that a good many of the assets of his portfolio have a purely local value and market. Finally, banks have and loan portfolios that are loans that have been made or bought and are being held for repayment. Loan portfolios are the major asset of banks, thrifts, and other lending institutions. The value of a loan portfolio depends not only on the interest rates earned on the loans, but also on the quality or likelihood that interest and principal will be paid.

1.2. Market risk and Value at Risk in a financial institution

1.2.1. Measuring market risk
There are significant differences in the internal and external views of what is a satisfactory market risk measure. Internally, bank managers need a measure that allows active, efficient management of the bank’s risk position. Bank regulators want to be sure a bank’s potential for catastrophic net worth loss is accurately measured and that the bank’s capital is sufficient to survive such a loss. Consider the differences in desired risk measure characteristics that these two views engender. Both managers and regulators want up-to-date measures of risk. For banks acting in trading, this may mean selective intraday risk measurement as well as a daily measurement of the total risk of the bank. Note, however that the intraday measures that are relevant for asset allocation and hedging decisions are measures of the marginal effect of a trade on total bank risk and not the stand-alone riskiness of the trade. Regulators, on the other hand, are concerned with the overall riskiness of a bank and have less concern with the risk of individual portfolio components. Nonetheless, given the ability of a sophisticated manager to “window dress” a bank’s position on
short notice, regulators might also like to monitor the intraday total risk. As a practical matter, they probably must be satisfied with a daily measure of total bank risk. The need of a total risk measure implies that risk measurement cannot be decentralized. For parametric measures of risk, such as standard deviation, this follows from the theory of portfolio selection (Markowitz, 1952) and the well-known fact that the risk of a portfolio is not, in general, the sum of the components risks. More general, imperfect correlation among portfolio components implies that simulations of portfolio risk must be driven by the portfolio return distribution, which will not be invariant to changes in portfolio composition. Finally, given costly regulatory capital requirements, choices among alternative assets require managers to consider risk return or risk trade-offs where risk is measured as the changing in portfolio risk resulting from a given change in portfolio composition. The appropriate risk scaling measure depends on the type of change being made. For example, the pertinent choice criterion for pure hedging transactions might be to maximize the marginal risk reduction to transaction cost ratio over the available instruments while the choice among proprietary transactions would involve minimizing marginal risk per unit of excess return.

Risk measurement is costly and time consuming. Consequently, bank managers compromise between measurement precision on the one hand and the cost and timeliness of reporting on the other. This trade-off will have a profound effect on the risk measurement method a bank will adopt. Bank regulators have their own problem with the cost of accurate risk measurement which is probably one reason they have chosen to monitor and stress test bank risk measurement systems rather than undertaking their own risk measurements.

Bank regulators have a singular risk measurement goal. They want to know, to a high degree of precision, the maximum loss a bank is likely to experience over a given horizon. They then can set the bank’s required capital to be greater than the estimated maximum loss and be almost sure that the bank will not fail over that horizon. In other words, regulators should focus on the extreme tail of the bank’s return distribution and on the size of that tail in adverse circumstances. Bank managers have a more complex set of risk information needs. In addition to shared concerns over sustainable losses, they must consider risk return trade-offs. That calls for a different risk measure than the “tail” statistic, a different horizon and a focus on more usual market conditions. Furthermore, even when concerned with the level of sustainable losses, the bank manager may want to monitor on the basis of a probability of loss that can be observed with some frequency (e.g. over a month rather than one year). This allows managers to use the risk measurement model to answer questions such as: Is the model currently valid? For example, if the loss probability is set at 5%, do we observe a violation once every 25 days on average? Are traders correctly motivated to manage and not just avoid risk? How often trader firsts position violate his risk limit relative to the likelihood of that event? What is the most I can lose on an investment? This is a question that almost every investor who has invested or is considering investing in a
risky asset asks at some point in time. Value at Risk tries to provide an answer, at least within a reasonable bound. In fact it is misleading to consider Value at Risk to be an alternative to risk adjusted value and probabilistic approaches. After all, it borrows liberally from both.

1.2.2. What is Value at Risk (VaR)?
Risk is unavoidable. Risk is the basic ingredient for generating profits in any market sensitivity activity. From the viewpoint of an investor, risk is about the odds of losing money, and VaR is based on that common sense fact. By assuming that investors care about the odds of a really big loss, VaR answers the question; “What is my worst-case scenario?” or “How much could I lose on a really bad month?”

In a few words, VaR is a single, statistical measure of possible portfolio losses. VaR shows the largest amount that a portfolio is likely to lose over a specific period of time at a specified level of confidence. For example, a “95% daily VaR” of €1 million would mean that the likelihood of that portfolio losing more than €1 million on the worst day is less than 5%. This in no way means that the portfolio cannot lose more than €1 million. In fact, over 100 days one would expect the portfolio to lose more than €1 million approximately 5 times. Furthermore, it does not mean that one could not cumulatively lose significantly more over a longer horizon. In its adapted form, the measure is sometimes defined more narrowly as the possible loss in value from “normal market risk” as opposed to total risk, requiring that we draw distinctions between normal and abnormal risk as well as between market and non-market risk.

While VaR can be used by any entity to measure its risk exposure, it is used most often by commercial and investment banks to capture the potential loss in value of their traded portfolios from adverse market movements over a specified period; this can be compared to their available capital and cash reserves to ensure that the losses can be covered without putting the firms at risk.

Taking a closer look at VaR, there are clearly key aspects, which are:
1. To estimate the probability of the loss, with a confident interval, we need to define the probability distributions of individual risks, the correlation across these risks and the effect of such risks on value.
2. To focus in VaR is clearly on downside risk and potential losses. Its use in banks reflects their fear of a liquidity crisis, where a low-probability catastrophic occurrence creates a loss that wipes out on the capital and creates a client exodus.
3. There are three key elements of VaR: a specified level of loss in value, a fixed time period over which risk is assessed and a confidence interval. The VaR can be specified for an individual asset, a portfolio of assets or for an entire firm.
4. While the VaR at investment banks is specified in terms of market risks (interest rate changes, equity market volatility and economic growth) there is
no reason why the risks cannot be defined more broadly or narrowly in specific contexts. Thus, we could compute the VaR for a large investment project for a firm in terms of competitive and firm-specific risks and the VaR for a gold mining company in terms of gold price risk.

4.2.3. History of VaR
The concept and use of VaR is not recent. VaR was first used by major financial firms in the late 1980s to measure the risks of their trading portfolios. J.P.Morgan was one of the first financial institutions to develop an internal VaR model. According to industry legend, their model is said to have originated when the chairman of J.P.Morgan, Dennis Weatherstone, asked his staff to prepare a daily one-page report, the famous “4:15 report”, indicating risk and potential losses over the next 24 hours, across the bank’s entire trading portfolio. The report was ready by around 1990 and the measure used was VaR, defined as the maximum likely loss over the next trading day.

Their model estimated VaR based on standard portfolio theory, using estimates of the standard deviations (volatilities) and correlations between the daily returns of different traded instruments. In October 1994, J.P.Morgan set up Risk Metrics as a spin-off company to establish a market standard through the release of their data and VaR methodology to third parties. This provided a tremendous impetus to the growth in the use of VaR and had a dramatic impact in raising risk management awareness worldwide.

Since that time period, the use of VaR had exploded. The subsequent adoption of VaR systems was very rapid, initially by securities houses and investment banks and later by commercial banks and other financial institutions. Currently VaR is used by the vast majority of financial institutions and is increasingly being used by smaller financial firms, institutional investors and non-financial corporations.

Regulators also became very interested in Value at Risk. In April 1995, the Basle Committee on Banking Supervision proposed allowing banks to calculate their capital requirements for market risk with their own VaR models, using certain parameters provided by the Committee. In June 1995, the US Federal Reserve proposed a “pre-commitment” approach which would allow banks to use their own internal VaR models to calculate capital requirements for market risk, with penalties to be imposed in the event that losses exceed the capital requirements. In January 1997, the US Securities and Exchange Commission adopted new rules for corporate risk disclosure (Securities Act Release No. 7386) which listed VaR as one of three possible market risk disclosure measures. The European Union’s Capital Adequacy Directive (CAD) which came into effect in 1996 allows VaR models to be used to calculate capital requirements for market risks. The Basel II framework, which comes into effect in 2007, continues to endorse internal VaR models to calculate capital requirements for
market risk and extends the potential use of “Credit VaR” models to calculate capital requirements for credit risk under the IRB-Advanced approach.

4.2.4. Variants of VaR
There are various extensions and variants of VaR measures, reflecting the range of applications and problems that the VaR framework can be used:

- **Incremental VaR**: this is a measure that gives us an indication of how the addition of a new position might impact the VaR of our portfolio. In other words, Incremental VaR is the change in VaR associated with adding a new position to our portfolio.

- **Marginal VaR**: this is a measure of how VaR changes if we increase the position by one additional unit of the underlying risk factor. In mathematical terms, Marginal VaR is the partial derivative of VaR with respect to each risk factor.

- **Cash Flow at Risk (CFaR) and Earnings at Risk (EaR)**: VaR describes a general class of probabilistic models that measure the risk of loss in market risk sensitive instruments and portfolios. However, the same mechanism can be applied to assess the uncertainty surrounding value in other settings as well, regardless of how value is defined (portfolio valuation, cash flow, or reported earnings). The metric that is actually being measured can be adapted to fit the circumstance in which the method is applied. Both are also probabilistic models developed from statistical analysis. CFaR is a reasonable choice for non-financial corporation’s which are concerned with managing the risks inherent in cash flows and not changes in mark-to-market values. Typically, the time horizon is much longer in CFaR calculations compared to Value at Risk. Many industrial companies already apply such tools to develop a more precise picture of their risk profile and make better hedging decisions.

4.2.5. Measuring VaR
There are three basic approaches that are used to compute VaR, though there are numerous variations within each approach. The measure can be computed analytically by making assumptions about return distributions for market risks and by running hypothetical portfolios through historical data or from Monte Carlo simulations.

**Variance Covariance Approach**
For the Variance Covariance approach, we employ one of the most widely used volatility estimation and forecasting methodology, the J.P.Morgan Risk Metrics. This approach applies exponentially declining weights to the returns from distant past (and greater weights to more recent returns) in order to estimate conditional volatilities and correlations. Exponential smoothing allows for cyclical behavior of return volatility to be captured. Exponentially weighted moving average model can be considered as an improvement over the traditional volatility forecasting method, which is based on moving averages with fixed, equal weights. Risk Metrics methodology assumes that:
- Returns on individual risk factors follow conditional normal distribution. While returns themselves may not be normally distributed and large outliers are far too common, the assumption is that the standardized return (computed as the return divided by the forecasted standard deviation) is normally distributed.
- The focus on standardized returns implies that is not the size of the return that we should focus on but its size relative to the standard deviation. In other words, a large return, positive or negative, in a period of high volatility may result in a low standardized return, whereas the same return following a period of low volatility will yield an abnormally standardized return.
- The change in position’s value is a linear function of the underlying return.

The focus on normalized standardized returns exposed the VaR computation to the risk of more frequent large outliers that would be expected with a normal distribution. In a subsequent variation, the Risk Metrics approach was extended to cover normal mixture distributions, which allow for the assignment of higher probabilities for outliers. Figure 1.1 contrasts the two distributions.

In effect, these distributions require estimates of the probabilities of outsized returns occurring and the expected size and standard deviations of such returns, in addition to the standard normal distribution parameters.

The strength of the Variance Covariance approach is that the VaR is simple to compute, once you have made an assumption about the distribution of returns and inputted the means, variances and covariances of returns. In the estimation process, though, lie the three key weaknesses of the approach:
• **Wrong distributional assumptions:** if conditional returns are not normally distributed, the computed VaR will understate the true VaR.

• **Input error:** the variance covariance matrix that is input to the VaR measure is a collection of estimates by using historical data, some of which have very large error terms.

• **Non-stationary variables:** a related problem occurs when the variances and covariances across assets change over time and that is not uncommon because the fundamentals driving these numbers do change over time.

• **Non linear assets are not allowed:** the Variance Covariance estimate of VaR is designed for portfolios where there is a linear relationship between risk and portfolio positions. Consequently, it can break down when the portfolio includes options, since the payoffs on an option are not linear.

**Historical Simulation**

Historical simulation is a simple, practical approach that requires virtually no assumption about the statistical distributions of the underlying risk factors. In essence, the approach involves using historical changes in market rates and prices to construct a distribution of potential future portfolio profits and losses and then reading off the VaR as the loss that is exceeded only 5% of the time (assuming 95% confidence). The distribution of profits and losses is constructed by taking the current portfolio and subjecting it to the actual percentage changes in the risk factors experienced during each of the last N periods. The use of the actual historical changes in rates and prices to compute the hypothetical profits and losses is the distinguishing feature of historical simulation.

Historical simulation can be described in terms of five steps:

**Step 1:** Identify the basic risk factors and obtain a formula expressing the mark-to-market value of the portfolio in terms of the risk factor.

**Step 2:** Obtain historical prices of the risk factors for the last N periods and calculate the corresponding percentage changes (returns) in these prices.

**Step 3:** Subject the current portfolio to the percentage changes calculated in Step 2, calculating the profits and losses that will occur if comparable returns in the risk factors are experienced and the current portfolio is marked-to-market.

**Step 4:** Order the mark-to-market profits and losses from the largest profit to the largest loss.

**Step 5:** From the ordered profits and losses select the loss which is equaled or exceeded 5% of the time (assuming 95% confidence level).

While historical simulations are popular and relatively easy to run, they do come with baggage. In particular, the underlying assumptions of the model generate give rise to its weaknesses:

• **Past is not prologue:** while all three approaches to estimating VaR use historical data, historical simulations are much more reliant on them for the simple reason that the VaR is computed entirely from historical price changes.
• **Trends in the data:** all data points in historical simulation are weighted equally. The price changes from trading days in a year affect the VaR in exactly the same proportion as price changes from trading days in another year. To the extent that there is a trend of increasing volatility even within the historical time period, the VaR will be understated.

• **New assets or market risks:** difficulty in dealing with new risks and assets for an obvious reason: there is no historic data available to compute the VaR.

The approach saves us the trouble and related problems of having to make specific assumptions about distributions of returns but it implicitly assumes that the distribution of past returns is a good and complete representation of expected future returns. In a market where risks are volatile and structural shifts occur at regular intervals, this assumption is difficult to sustain.

**Monte Carlo Simulation**

Monte Carlo simulation is a general problem solving technique used to approximate the probability of certain outcomes by running multiple trial runs, called simulations, using random variables. It has a number of similarities to historical simulation. The main difference is that rather than carrying out the simulation using the observed prices in the risk factors over the last N periods to generate N-1 hypothetical portfolio profits and losses, one chooses a statistical distribution that is believed to adequately capture or approximate the possible changes in the risk factors. Then, a pseudo-random number generator is used to generate thousands or perhaps tens of thousands of hypothetical changes in the risk factors. These are then used to construct thousands of hypothetical profits and losses on the current portfolio. Finally, the VaR is then determined from the distribution of possible portfolio profits and losses.
Monte Carlo simulation can be described in terms of five steps:

*Step 1:* Identify the basic risk factors and obtain a formula expressing the mark-to-market value of the portfolio in terms of the risk factor.

*Step 2:* Determine a specific distribution (stochastic process) for the returns in the basic risk factors and to estimate the parameters of that distribution. The ability to pick the distribution is the feature that distinguishes Monte Carlo simulation from the other two approaches.

*Step 3:* Use a pseudo-random generator to generate $N$ hypothetical values of changes in the risk factors, where $N$ is sufficiently large. These hypothetical risk factors are then used to calculate $N$ hypothetical mark-to-market portfolio values. Then, from each of the hypothetical portfolio values we subtract the actual mark-to-market portfolio value to obtain $N$ hypothetical profits and losses.

*Step 4 & 5:* The two last steps are the same as in historical simulation. The mark-to-market profits and losses are ordered from the largest profit to the largest loss and the VaR is the loss which is equaled or exceeded 5% of the time.

The designers of a risk management system are free to choose any distribution that they think reasonably describes possible future changes in the risk factors. Beliefs about future changes are typically based on observed past changes, so this amounts to saying that the designers are free to choose any distribution of past changes in the risk factors. However, Monte Carlo simulation has the advantage of allowing users to tailor ideas about future patterns that depart from historical patterns. On the other hand, as the number of market risks increases and their co-movements become more complex, Monte Carlo simulation becomes more difficult to run for two reasons. First, the estimation of the probability distributions for hundreds of market risk variables rather
than just the handful of analyzing a single project or asset. Second, the number of simulations that are needed to run to obtain reasonable estimate of VaR will have to increase substantially.

4.2.6. **Comparison of VaR methodologies**

With three methods from which to choose, the obvious question is: “Which method of calculating VaR is best”? Unfortunately, there is no easy answer. Each one of the three approaches to estimating VaR has advantages and comes with baggage. The different methods of calculating VaR differ in their ability to capture the risks options and option-like instruments, ease of implementation, ease of explanation to senior management, flexibility in analyzing the effect of changes in the assumptions and reliability of the results. The best choice will be determined by which dimensions the risk manager finds most important.

In table 1.1 we can see a comparison between the three models.

<table>
<thead>
<tr>
<th><strong>Comparison of VaR methodologies</strong></th>
<th>Variance and Covariance</th>
<th>Historical Simulation</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Able to capture the risks of portfolios which include options?</strong></td>
<td>No, except when computed using a short time period for portfolios with limited or moderate options content</td>
<td>Yes, regardless of the options content of the portfolio</td>
<td>Yes, regardless of the options content of the portfolio</td>
</tr>
<tr>
<td><strong>Ease to implement?</strong></td>
<td>Yes, depending upon the complexity of the instruments and availability of data</td>
<td>Yes, for portfolios for which data on the past values of the risk factors are available</td>
<td>Yes, depending upon the knowledge and technical expertise of the user</td>
</tr>
<tr>
<td><strong>Computations performed quickly?</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>No, except for relatively small portfolios</td>
</tr>
<tr>
<td><strong>Easy to explain to senior management?</strong></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Produces misleading VaR estimates when recent past is not typical?</strong></td>
<td>Yes, except that alternative standard deviations/correlations may be used</td>
<td>Yes</td>
<td>Yes, except that alternative estimates of parameters may be used</td>
</tr>
<tr>
<td><strong>Easy to perform “what-if” analysis to examine effect of alternative assumptions?</strong></td>
<td>Easily able to examine alternative assumptions about standard deviations/correlations. Unable to examine alternative assumptions about</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
4.2.7. **Ways VaR can be used**

VaR provides a forward-looking analysis of the portfolio’s risk profile in a comprehensive and consistent fashion. In order to maximize the benefits of having a VaR system in place, it should be used not only for risk measurement, but also as a:

- **Decision-making tool**: VaR creates a common denominator with which to compare various risky choices. Having exposed the risk profile of all alternative portfolio choices within a VaR framework, we are able to decide which option suits better our risk preferences. Making risk-based decisions is a way to use the information provided by the VaR analysis more effectively.

- **Risk allocation tool**: VaR can be used as a guide to allocate risk capital within an organization through setting VaR-based position limits. By introducing VaR-based position limits, we are able to allocate risk capital much more efficiently within the firm. The total VaR limit can be broken down into smaller components. The level at which the limit is set for each unit should represent an allocation of capital reflecting that unit’s appetite for risk tempered by the firm’s total risk tolerance.

- **Performance evaluation tool**: VaR can be used as a tool for risk-adjusted performance evaluation. It gives us the ability to measure the performance of each trader, we have a better yardstick, or to compare the performances between two or more traders.

4.2.8. **Limitations of VaR**

While VaR has acquired a strong following in the risk management community, there is reason to be skeptical of both its accuracy as a risk management tool and its use in decision making. The reasons are:

- **VaR can be misleading**: there is no precise measure of VaR and each measure comes with each own limitations. The end result is that the VaR that we compute for an asset, portfolio or a firm can be wrong, and sometimes, the errors can be large enough to make VaR a misleading measure of risk exposure. For example, we might have two positions with equal VaRs at some confidence level and holding period and yet one position might involve much heavier “tail” losses than the other. The VaR measure taken on its own would incorrectly suggest that both positions were equally risky. This is perhaps the greatest limitation of VaR, its lack of information on tail losses.

- **True VaR can be much greater than computed VaR**: VaR measures the likelihood of losses to an asset or portfolio due to market risk. First, risk is almost always considered to be a negative in VaR. While there is no technical
reason why one cannot estimate potential profits that one can earn with 99% probability, VaR is measures in terms of potential losses and not gains. Second, most VaR measures are built around market risk effects, while there is no reason why we cannot look at the VaR, relative to all risks, practicality forces up to focus on just market risks and their effects in value.

- **Overexposure to Risk:** even if VaR is correctly measured, it is not clear that using it as the measure of risk leads to more reasonable and sensible decisions on the part of managers and investors. In fact, there are two strands of criticism against the use of VaR in decision making. The first is that making investment decisions based upon VaR can lead to overexposure risk, even when the decision makers are rational and VaR is estimated precisely. The other is that managers who understand how VaR is computed, can game the measure to report superior performance, while exposing the firm to substantial risks.

### 4.2.9. Supplementary risk measures

**Stress testing**

The limitations of VaR revealed the need for supplementary risk measures. VaR is a single, summary, statistical measure of risk under normal market conditions, but markets do not always behave as expected and unanticipated “shocks” do occur from time to time. When this happens (or rather before it happens), it is important to ask “What might be the potential impact of a market crisis on my portfolio?” or “If my VaR is exceeded, just how much of a loss can my firm sustain before it goes out of business?”

Stress testing attempts to answer these questions by performing a set of scenario analysis to investigate the effects of extreme market conditions. To the extent that the effects are unacceptable, the portfolio or risk management strategy needs to be revised. There is no standard way to carry out stress testing and no standard set of scenarios to consider. Rather, the process depends crucially on the judgment and experience of the risk manager.

There are two types of stress tests that can be performed. The first is actual crisis historic scenarios, which has the benefit that it preserves the embedded historical correlation effects between markets and instruments when crisis happened. The ability of stress tests to replicate this phenomenon is critical, because correlations across risk factors tend to converge to one (highly correlated) in times of stress. The second type of stress testing is stress scenarios. These are defined by the user and generally represent significant step moves in the market.

After developing a set of historic or stress scenarios, the next step is to determine the effect of the assumed market shocks on the prices of all assets in the portfolio and the impact on portfolio value. In addition, companies whose risk management strategy depends upon the ability to frequently adjust or rebalance their portfolios need to consider the impact of major surprises on market liquidity. It may be difficult or
impossible to execute transactions at reasonable bid/ask spreads during periods of market stress.

**Expected Tail Loss**

There is one question, “When the VaR is exceeded how large can the losses be?”, that VaR cannot answer. However, there is one measure which can supplement VaR to provide the information that is missing on tail losses. This is the expected Tail Loss, which is the expected value (average) of our losses, $L$, if we get a loss in excess of VaR. in mathematical terms:

$$E[L | L > VaR]$$

In other words, the expected Tail Loss is estimated by averaging out all the losses exceeding VaR from the distribution of potential future portfolio losses and profits computed by the VaR model. Obviously, both VaR and expected Tail Loss depend on the same underlying parameters and distributional assumptions. However, VaR tells us the most we can expect to lose if a tail event does not occur and the expected Tail Loss what we can expect to lose if a tail event does occur.

**CHAPTER 2: Portfolio VaR and Systematic Risk**

**4.3. Investment theories and VaR analysis**

**4.3.1. History of the modern portfolio theory**

The main investment tools in modern portfolio theory are:

- Markowitz Theory
- Tobin’s Separation Theorem
- Capital Asset Pricing Model (CAPM)
- Capital Market Line (CML)
- Security Market Line (SML)
- Single Index Model (SIM)
- Arbitrage Pricing Theory (APT)

Things really began with Harry M. Markowitz and that active risk management will be able to develop to the point of becoming an essential element in portfolio management. 1952: Publication of an article “Portfolio Selection” in the "Journal of Finance". 1959: Publication of "Portfolio Selection: Efficient Diversification of Investments". Harry Markowitz formalized what investors already knew when they looked to have placement profitability correspond to the level of risk. But he was the first person to mathematically establish that the total risk of a portfolio is inferior to the sum of the individual risk for each element of a portfolio. By taking periodical performances as random variables, it was possible to calculate performance expectations, standard deviation and correlations. By seeking the minimum risk for each level of performance, we obtain what Markowitz called: the efficient frontier. 1958: James Tobin extends the field of studies by introducing the risk free asset. It is therefore possible to study the cases where the investor becomes the lender or borrower and the impact of financial levers. This will bring the concept of “Capital Market Line” and allow filing in line, all of the portfolios made up of a combination of risky portfolios and risk free asset.
1964: William F. Sharp formalized the "Capital asset pricing model" (CAPM). By introducing the β (beta) and using the CAPM, Sharp establishes a simple relation to obtain expected returns without having to go through the manipulation of large numbers of data and in particular the calculation of co-variances.

1976: Stephen A. Ross introduces the "Arbitrage Pricing Theory" or the model of evaluation by arbitrage (MEA).

The CAPM remained a model of reference until the beginning of the 1980's. But Stephen Ross tried to get rid of the restrictive hypothesis of the CAPM by elaborating a new model in which the risk factors are many.

4.3.2. Markowitz Theory

The Markowitz framework is often generically known as the mean-variance framework. The assumptions of this model are:

- Investors base their decisions on expected return and risk, as measured by the mean and variance of the returns on various assets.
- All investors have the same time horizon. In other words, they are concerned only with the utility of their terminal wealth and not with the state of their portfolio beforehand and this terminal time is the same for all investors.
- All investors are in agreement as to the parameters necessary, and their values (information is freely and simultaneously available to all market participants), in the investment decision making process, namely, the means, variances and correlations of returns on various investments. The investors are homogeneous.
- Financial assets are arbitrarily fungible.

The basic tenant of the Markowitz theory is that knowing the mean and standard deviation of the returns on the portfolio is sufficient and that our desire is to maximize the expected return and to minimize the standard deviation of the return. The standard deviation is the measure of riskiness of the portfolio. One thing that is obvious, because individuals are utility maximizers, is that they will always switch from one investment to another which has the same expected return but less risk, or one which...
has the same risk but greater expected return, or one which has both greater expected return and less risk. (Figure 2.1)

4.3.3. Tobin’s Separation Theorem

Ever since the 1957 publication of James Tobin's seminal paper on what has become known as "The Separation Theorem", investors have had the theoretical basis for modulating portfolio risk.

In essence, the Theorem postulates that an investor can control the risk of a basket of risky investments by either borrowing at the risk free rate or leveraging the portfolio (and its risk), or alternately, lending at the risk free rate and tempering risk. Since most investors are risk averse, the clear preference of most investors is to combine the risky basket of securities with risk free bonds, and thereby lower the downside risk of the portfolio. In common parlance, we would term this the stock/bond asset allocation decision.

The risk free bond allocation is usually defined as short-term treasury securities (1 to 5 year maturities), or perhaps expanded to include short term investment grade corporate paper. It is sometimes suggested that the risk free allocation should be made up of inflation indexed treasuries.

James Tobin, said if you hold risky securities and are able to borrow, buying stocks on margin, or lend, buying risk-free assets, and you do so at the same rate, then the efficient frontier is a single portfolio of risky securities plus borrowing and lending, and that dominates any other combination. (Figure 2.2)

Tobin's Separation Theorem says you can separate the problem into first finding that optimal combination of risky securities and then deciding whether to lend or borrow, depending on your attitude toward risk. It then showed that if there's only one portfolio plus borrowing and lending, it's got to be the market.

4.3.4. Capital Asset Pricing Model (CAPM)

The CAPM is what is known as an equilibrium model. The market participants act to put the market into equilibrium. A number of additional assumptions (over and above those of Markowitz) are made in the CAPM, which are thought to be not too far from
reality, yet are useful in order to simplify the derivation of the model. Of course, a set of such assumptions is necessary in any economic model. In this model, they are:

- Unlimited short sales are allowed.
- There is a risk free rate \( R_f \) for lending and borrowing money. The rate is the same for lending and borrowing and investors have any amount of credit.
- There are no transaction costs in the buying and selling of capital assets.
- Similarly, there are no income or capital gains taxes.
- The market consists of all assets. (No assets are exclusively private property).
- Assets are infinitely divisible.
- Perfect competition--an individual cannot affect the price of an asset by his/her buying or selling.
- All investors have the same information.
- Investors make their decisions based solely on the expected returns and variances of portfolio returns.

The expected return on an asset can be divided into two parts, the return for deferring consumption, and a compensation for bearing risk. The return for deferring consumption is the return on the risk-free asset. Hence the return for bearing risk is \( E(R) - R_f \). Mean-variance analysis implies that the return for bearing risk is proportional to the risk. Consequently, in mathematical terms CAPM is:

\[
E(R) = R_f + \beta \times (E(R_m) - R_f)
\]

Where, \( \beta \) is the risk premium, which actually is the return in excess of the risk free rate of return that an investment is expected to yield and \( E(R_m) \) is a parameter called beta, which is used to describe how well a security or portfolio correlates to the return of the market as a whole.

There are three areas of interest:
1. \( \beta = 0 \): An asset that has no volatility (no risk) does not have returns that vary with the market and therefore has a beta of zero and an expected return equal to the risk-free rate.
2. \( \beta = 1 \): An asset that moves with a volatility exactly equal to the market has a beta of one. In other words, it is perfectly correlated. By definition, its return rate is equal to the market. \( E(R) = E(R_m) \)
3. \( \beta > 1 \): An asset that experiences greater swings in periodic returns than the market, which, by definition, has a beta greater than one. This asset is expected to earn returns superior to those of the market as compensation for this extra risk.

The general idea of CAPM is that investors should be compensated in two ways: time value of money and risk. An asset is expected to earn the risk-free rate plus a reward for bearing risk as measured by that asset’s beta. Figure 2.3 below demonstrates this predicted relationship between beta and expected return, this line is called the Security Market Line.
For example, a stock with a beta of 1.5 would be expected to have an excess return of 15% in a time period where the overall market beat the risk free asset by 10%.

The CAPM model is used for pricing an individual security or a portfolio. For individual securities, the security market line (SML) and its relation to the expected return and systematic risk (beta) shows how the market must price individual securities in relation to their security risk class.

4.3.5. **Capital Market Line (CML)**

Capital market line is referred to as a measure employed to evaluate portfolio performance. Capital market line or CML is a graph employed in asset pricing models to depict rates of return in a market portfolio. Capital market line describes rates of return for efficient portfolios that are dependent on level of risk and risk free rate of return for a specific portfolio. CML originates from the assumption that all investors will possess market portfolio. Quantum of risk is positively correlated to the expected return.
Capital market line, Figure 2.4, is deduced by drawing a tangent line that starts from the intercept point located on efficient frontier and extends to the point where expected return matches risk free rate of return. Capital market line is believed to be a better measure than efficient frontier as it takes into consideration risk free asset in a portfolio. All points on the CML have better risk return profiles when compared to any portfolio located on efficient frontier.

4.3.6. Security Market Line (SML)
The Security Market Line can be thought of as the graphical representation of the Capital Asset Pricing Model. It illustrates the concept that it is possible to obtain any combination of risk and expected return along the slope of the graph by investing some portion of your investment in the market portfolio and borrowing the rest.

The Security Market Line is useful for determining whether an investment in an asset offers a good expected return for the risk taken. By providing the Beta of the asset, the Risk free rate and the Market Risk Premium, we will be able to plot the asset on the
Security Market Line graph (Figure 2.5). If the Expected return versus Beta of the asset is plotted above the Security Market Line, the asset can be thought of as being able to provide a greater return for the inherent risk. An asset with a point below the Security Market Line can be thought of as getting less return for the amount of risk taken.

4.3.7. Single Index Model (SIM)
The Single Index Model assumes that there is only one macroeconomic factor that causes the systematic risk affecting all stock returns and this factor can be represented by the rate of return on a market index. According to this model, the return of any stock can be decomposed into the expected excess return of the individual stock due to firm-specific factors, commonly denoted by its alpha coefficient ($\alpha$), the return due to macroeconomic events that affect the market, and the unexpected microeconomic events that affect only the firm. Specifically, the return of stock $S$ is:

$$ r_S = \alpha + \beta r_M + \epsilon $$

The term $\alpha$ represents the stock's return due to the movement of the market modified by the stock's beta, while $\beta r_M$ represents the unsystematic risk of the security due to firm-specific factors.

Macroeconomic events, such as interest rates or the cost of labor, causes the systematic risk that affects the returns of all stocks, and the firm specific events are the unexpected microeconomic events that affect the returns of specific firms, such as the death of key people or the lowering of the firm's credit rating, that would affect the firm, but would have a negligible effect on the economy. The unsystematic risk due to firm specific factors of a portfolio can be reduced to zero by diversification.

The Single Index Model is based on the following:

- Most stocks have a positive covariance because they all respond similarly to macroeconomic factors.
- Some firms are more sensitive to macroeconomic factors than others and this firm specific variance is typically denoted by its beta, which measures the variance compared to the market for one or more economic factors.
- Covariances among securities result from differing responses to macroeconomic factors. Hence, the covariance of each stock can be found by multiplying their betas and the market variance.

This last equation greatly reduces the computations required to determine covariance because the covariance of the securities within a portfolio must be calculated using historical returns, and the covariance of each possible pair of securities in the portfolio must be calculated independently. With this equation, only the betas of the individual securities and the market variance need to be estimated to calculate covariance. Hence, the index model greatly reduces the number of calculations that would otherwise have to be made for a large portfolio of thousands of securities.

4.3.8. Arbitrage Pricing Theory (APT)
Arbitrage pricing theory (APT), in finance, is a general theory of asset pricing, which has become influential in the pricing of stocks. APT holds that the expected return of a
financial asset can be modeled as a linear function of various macro-economic factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient. The model-derived rate of return will then be used to price the asset correctly, the asset price should equal the expected end of period price discounted at the rate implied by model. If the price diverges, arbitrage should bring it back into line. The theory was initiated by the economist Stephen Ross in 1976.

The fundamental foundation for the arbitrage pricing theory is the law of one price, which states that two identical items will sell for the same price, for if they do not, then a riskless profit could be made by arbitrage, buying the item in the cheaper market then selling it in the more expensive market. It is predicated on the fact that two financial instruments or portfolios, even if they are not identical, should cost the same if their return and risk is identical. The justification for this is that the only reason that a financial instrument is purchased is to earn a return for a certain amount of risk, no other aspect of the financial instrument matters. Hence, the law of one price requires that any two financial instruments or portfolios that have the same return-risk profile should sell for the same price. If this is not true, then a profit could be made by selling short the security or portfolio with the lower return, and buying the higher return portfolio. The assumptions of the arbitrage pricing theory are:

- Investors seek return tempered by risk: they are risk averse and seek to maximize their terminal wealth.
- There is a risk free rate for lending and borrowing money.
- There are no market frictions.
- Investors agree on the number and identity of the factors that are systematically important in pricing assets.
- There are no riskless arbitrage opportunities.

The model starts with a linear equation:
Where, the value at time $t$ of index $j$, $s$ the index for a single security, the return on the single security $s$, the $a$-parameter of security $s$, the sensitivity of the return on security $s$ to the level of index $j$ and a random variable.

The APT appears to be a multi-factor version of the CAPM model where the returns are sensitive to the levels of indices, rather than to the returns of the single index. Typical APT indices include:

- Unanticipated changes in inflation.
- Unanticipated changes in industrial production.
- Unanticipated changes in risk premium, as measured in corporate bond spreads.
- Unanticipated changes in the slope and level of the term structure of interest rates.

The APT theory involves a derivation of an equilibrium model, via an assumption of homogeneous expectations.

4.4. Beta Coefficient

4.4.1. Beta Coefficient and Systematic Risk

The beta coefficient is a measure of an asset's risk and return in relation to a broad market, meaning that it will show, more or less, how the asset or a portfolio of assets will respond as the market moves up or down. It is used in the CAPM and regression analysis.

A beta coefficient will show how an asset's performance is sensitive to systematic risk, which is the risk that can affect an entire market. An investor who is seeking to measure the expected return of a particular stock, for example, will use a stock market index to represent the broad market. The stock market index will normally have a beta coefficient of 1, and in theory, a security whose beta is 1.4, for example, will move 1.4 times the move of the index. This means that if the stock market index was to move up or down by 20 percent, the security would move 28 percent accordingly.

On average, many securities have a beta coefficient of 1, which means that they move more or less in line with the market. A security with a beta coefficient of more than 1 is more risky than the average market and is fit for more aggressive investment strategies. On the other hand, those whose beta coefficient is below 1 are considered to be less risky, because their performance is less tied to the systematic risk. Moreover, there are assets whose beta is negative, and these tend to have dull returns when the economy is robust, but in a downturn, they have a tendency of outperforming most other investments.

The asset with a negative beta is inherently less sensitive to systematic risk, and for this reason, an investor might use this type of asset to hedge his or her portfolio. To hedge, in this sense, is to try to offset losses that might result if a systematic event arises. Moreover, when performing a regression analysis, an individual might use historical data of returns in order to estimate the link between an asset's performance and that of the wider market.
The beta of an asset can change over time; for example, the beta of a particular asset can be 1.2 for about a decade, then for various reasons, it might change to 1.4 in the following decade. Thus, in regression analysis, the beta coefficient is meant to be the same for the period being sampled. That is, if an individual was to use a sample from two decades where in one it was 1.2 and the other 1.4, the resulting information will most likely be misleading.

4.4.2. Methods of estimating Beta

Beta is simply a measure of sensitivity of stock to market movement. There are three methods of estimating beta as forecasters of covariance:

- **Unadjusted beta:** the first method simply estimates beta from historical data. The historical beta for each stock S can be obtained through regression analysis of stock return against market return from a past period, t=1 to t=T. The calculation of beta for each stock is formally shown below. The estimation of historical beta is subjected to error and might deviate significantly from actual beta since actual beta is not perfectly stationary over time. The betas might change significantly from one period to another and large random error may lead to substantial forecasting error.

- **Blume’s beta:** Blume’s analysis on the behavior of betas over time shows that there is a tendency of actual betas in the forecast period to move closer to one than the estimated betas from historical data. Blume’s technique attempts to describe this tendency by correcting historical betas to adjust the betas towards one, assuming that adjustment in one period is a good estimate in the next period. Consider betas for all stocks S in period 0, and betas for the same stocks S in the successive period 1. The betas for period 1 are then regressed against the betas for period 0 to obtain the following equation: . The relationship implies that the beta in period 1 is k1+k2 times the beta in the period 0. Therefore, if β is A, the estimate of beta in the next period will be (k1+k2*A) instead of A. This adjustment sets the average beta to undergo similar trend for subsequent forecast periods. If there is an increasing beta for period 1, average beta for period 2 will consequently increase. This might not reflect the actual beta movement from one period to another. Hence, Blume further modified the average beta towards historical mean. This was done by first calculating the average beta of all stocks for period 1 and 2, and . To adjust the mean of the forecasted beta towards historical mean, the new forecast of beta for each stock S is obtained by subtracting from the previously forecast of beta and adding.

- **Vasicek’s beta:** as mentioned earlier, the average beta tends to move towards one over time. Another method to capture this tendency is via Vasicek’s technique. Vasicek’s technique adjusts past betas towards the average beta by modifying each beta depending on the sampling error about beta. When the sampling error is large, there is higher chance of larger difference from the
average beta. Therefore, lower weight will be given to betas with larger sampling error. The following formula demonstrates this idea:

4.4.3. Beta stability: Fabozzi and Francis/Kim and Zumwalt/Chen research

Over the past four decades, many studies have theoretically and empirically examined the validity of the CAPM developed by Sharpe (1964), Lintner (1965) and Black (1972). The CAPM indicates that the systematic risk or beta is only variable that can explain the difference in average returns between stocks. While the CAPM is an extremely elegant and useful tool, many researchers have doubt on the overall efficiency of the model.

Recently, a number of tests on the CAPM model have been examined. Many studies document a consistent and highly significant relationship between average return and beta based on the assumption of constant risk. However, others evidence that beta alone has less ability in explaining the returns because firm size can play an important role, in addition to the beta.

In addition, there are concerns about the varying risks that become main factors in return-generating process. Various studies indicate that the CAPM model assuming constant risk cannot participate in Bull and Bear market conditions. Levy (1971) suggests that there is a need to separate betas between Bull and Bear markets. However, Fabozzi and Francis (1977) propose the varying risk model to examine the stability of beta over these two markets. They find that the beta is stable even the market conditions change. Later on, many empirical studies support Levy (1971) that there is a need of calculating two betas, one for Bull period and the other for Bear period. It is because the traditional CAPM does not work well when market conditions change.

Fabozzi and Francis (1977, 1979) indicate that the CAPM show significant results in Bull and Bear market periods. They are among the first to evidence the stability in betas over Bull and Bear market on individual stocks and mutual funds. Fabozzi and Francis (1977) use a sample of 700 NYSE stocks and examine whether the beta in CAPM model differs significantly when measured over Bull and Bear markets during 1966-1971. They indicate that the betas appear to be insignificantly affected by the change in conditions between the two periods. Furthermore, Fabozzi and Francis (1979) study whether betas are persistent for mutual funds from 1965 to 1971. They indicate that mutual funds generally respond indifferently to Bull and Bear markets. These empirical results reinforce that mutual fund managers do not raise their betas during the Bull periods and do not reduce their betas during the Bear periods to earn additional risk-adjusted premiums. Eventually, these imply no difference in beta during Bull and Bear markets. Since the beta does not differ with market conditions, the use
of beta estimated for the entire period (constant beta) is still powerful in return-generating process.
Kim and Zumwalt (1979) argue on the finding of Fabozzi and Francis (1977, 1979) in which the betas of the CAPM model are sustainable in both Bull and Bear markets. They oppose that even if the constant betas are potentially sound in the two types of markets, there should be the return variations in Bull market that may not consistent with those in Bear market. Rather, they believe that the risk-averse investors would demand higher risk premium when taking unfavorable risk during the Bear market and pay a premium when consuming favorable risk during the Bull market. Therefore, they examine the risk premiums associated with the returns variation in these two markets during 1962-1976. They find the positive risk premium in Bear periods and the negative premium in Bull periods. Thus, they suggest that the responses to Bull and Bear markets allow betas to vary over time.
Consistent with Kim and Zumwalt (1979), Chen (1982) also shows that the varying risks in market model appears to be more appropriate than the constant risks when the Bull and Bear conditions are taken into account. Using the Kim and Zumwalt (1979) procedures, he finds that during 1965-1977, investors prefer higher compensation if assuming the unfavorable variations of returns occur in Bear markets. Investors then pay premium for the favorable variations of return occurring in Bull markets.

4.4.4. Investors attitude towards risk
Investors’ attitude towards risk varies. Some investors are willing to take on greater risk in the hope for greater returns while other investors are less willing to take on risk. Investors can therefore be classified in three categories:

- **Risk averse:** most investors are risk averse, that is, they do not like risk. If an investor has the choice between two investments with the same expected return but different levels of risk, they will choose the one with the lowest risk. If an investor has the choice between an investment with different expected returns and the same level of risk, they will choose the one with the highest expected return. What happens if the two investments have differing returns and risk? It will depend on how risk averse the investor is. If an investor is really risk averse then the choice is the investment with the lowest risk.

- **Risk neutral:** risk neutral investors do not concern themselves with risk. They will seek the highest expected return regardless of the risk involved. Their choice depends only on returns and has nothing to do with risk.

- **Risk seeker/lover:** hence as the name suggests, a risk seeking investor will always choose the investment with the highest risk regardless of return. Risk seekers actively seek out risky investments.

Since most investors are risk adverse financial markets behave as if, collectively, they are risk averse. This makes sense as investor control the financial markets and if the majority of them as a whole are risk averse then financial markets are expected to act the same way.
CHAPTER 3: Empirical Illustration

4.1. Collecting and analyzing data
In order to complete the empirical illustration we need to collect the data. We use daily closing prices of five Greek bank stocks (ATE bank, Alpha bank, Emporiki, Eurobank and Piraeus) for a period of five years and the daily prices for the same period of the Greek stock index (ASE). This index is the one that shows the general movement of the Greek stock exchange market, it is a weighted index and involves over 60 high capital corporations, so we can say that it is a representative indicator.

Furthermore, we assume that we have a portfolio that includes those five bank stocks and that we will invest in this portfolio 5,000.00€.

Asymmetry of Returns
Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point. In all of the stocks we can see that skewness (ε) is higher than zero, so time series are asymmetry to the right. Kurtosis on the other hand is a measure of whether the data are peaked or flat relative to a normal distribution. That is, data sets with high kurtosis tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. A uniform distribution would be the extreme case. And because kurtosis (κ) for all stocks is different from 3 (κ) we do not have normal distribution. More specifically, kurtosis is actually higher than 3 (>3) so returns tend to have a distinct peak near the mean.

Expected Returns
In order to calculate the expected returns of the prices of our stocks we have used the logarithmic returns:

Why use logarithmic returns? By using log prices we can convert an exponential problem to a linear problem. Logarithmic returns are simply first differences of log prices sampled at the same unit time interval. Sums of logarithmic returns over a time interval, give the logarithmic return for that interval, and a mean return can be calculated by dividing that interval by the number of time units in the interval.

Note that the mean log return of a time series of prices is determined by only three numbers, the starting price, the last price and the number of time intervals between the prices. This is true even if the series consisted of thousands of returns. This "expected" return thus gives no information about what happened over the sub intervals. On a log price plot this is the same as determining the rate by fitting the price series by drawing a line through the first and last prices.

Regression of stocks in Greek index
To run a regression we have used the Ordinary Least Squares (OLS) regression is a method for estimating the unknown parameters, in our case the beta and the alpha of the regression, as it is described in the single index model:

Also coefficient correlation of the stock with the index is:

The characteristic line shows the changes in returns of every stock and the index. The slope is the beta and is measures in ratio scale.
We have run a regression; by using excel, in order to estimate the betas. Regression equation:

*Estimations with OLS*
We use Kim & Zumwalt research to investigate if there is any asymmetry in beta coefficient, according to the equation:

With,

Now, we set that D=0 so:

We run the regression for each stock and we have collected the following data:
The results are:

<table>
<thead>
<tr>
<th></th>
<th>ATE</th>
<th>ALPHA</th>
<th>EMPORIKI</th>
<th>EUROBANK</th>
<th>PIREUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0.067689</td>
<td>1.20141</td>
<td>1.19097</td>
<td>1.07211</td>
<td>1.13520</td>
</tr>
<tr>
<td></td>
<td>0.105538</td>
<td>0.48</td>
<td>0.419</td>
<td>0.567</td>
<td>0.528</td>
</tr>
<tr>
<td></td>
<td>0.60212</td>
<td>1.27167</td>
<td>1.30613</td>
<td>1.15298</td>
<td>1.19162</td>
</tr>
<tr>
<td></td>
<td>0.75248</td>
<td>1.13138</td>
<td>1.07453</td>
<td>0.99034</td>
<td>1.07815</td>
</tr>
<tr>
<td></td>
<td>0.10610</td>
<td>0.48</td>
<td>0.421</td>
<td>0.568</td>
<td>0.529</td>
</tr>
</tbody>
</table>

Table 3.1.

From function (1) the expected returns and the variance of each stock is:

As we can see from table 3.1 for the four stocks out of five

While the opposite, occurs only in the stock of ATE bank. After running the regression we can see that in our case the beta of our portfolio rises when the market goes down and vice versa.

*Portfolio VaR Estimation with “beta model”*
We use the function:
We can observe, as it is shown in table 3.2, that by using “beta model”:

<table>
<thead>
<tr>
<th>Stock</th>
<th>s.d.</th>
<th>VaR 95%</th>
<th>VaR 99%</th>
<th>Systematic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATE</td>
<td>0,022125</td>
<td>36.395€</td>
<td>51.464</td>
<td>0,000490</td>
</tr>
<tr>
<td>ALPHA</td>
<td>0,018413</td>
<td>30.289€</td>
<td>42.834</td>
<td>0,000339</td>
</tr>
<tr>
<td>EMPORIKI</td>
<td>0,019516</td>
<td>32.103€</td>
<td>45.399</td>
<td>0,000381</td>
</tr>
<tr>
<td>EFG</td>
<td>0,015118</td>
<td>24.869€</td>
<td>35.169</td>
<td>0,000229</td>
</tr>
<tr>
<td>PIREUS</td>
<td>0,016578</td>
<td>27.269€</td>
<td>38.564</td>
<td>0,000275</td>
</tr>
<tr>
<td>INDEX</td>
<td>0,0106188</td>
<td>87,34</td>
<td>123,512</td>
<td>0,000113</td>
</tr>
<tr>
<td></td>
<td>0,006387688</td>
<td>52,54</td>
<td>74,298</td>
<td>0,000040802</td>
</tr>
<tr>
<td></td>
<td>0,006338766</td>
<td>52,14</td>
<td>73,729</td>
<td>0,000040180</td>
</tr>
</tbody>
</table>

Table 3.3

Where systematic risk is calculated by using the following formula:
CONCLUSION

In the empirical part, it was found the impact of beta coefficient instability, that it is observed in upward and downward market conditions, in evaluating VaR of a single stock but and of a portfolio. Even though VaR seems to be underestimated by using beta coefficient, that method of calculating Value at Risk combined with the volatility of systematic risk gives us the opportunity to observe a high correlation of the market with risk management. The quantification of market risk for bank portfolios can be differentiated depending on the impact of markets in systematic risk and on the differentiated level degree of the portfolios. So it is bank managers’ decision whether or not they will reconstruct their portfolios in order to be in the desirable level of risk. The economic crises in global markets led in the creation of new more sophisticated models in order to manage risk. And as the story goes, the need of new ideas and solutions will appear much more often.

REFERENCES


